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LEARNING AS REORGANIZATION

An Experimental Study in Third-Grade Arithmetic

BY

WILLIAM A. BROWNELL

WITH THE ASSISTANCE OF

KENNETH G. KUEHNER AND WILLIAM C. REIN

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Miss Elizabeth Boyd
Miss Chapella Dunlap
Miss Jennie Whisnant
(co-ordinator)
Mrs. Louise Williamson

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Miss Lucille Boone
Miss Virginia Harrison
(co-ordinator)
Mrs. Dorothy Ljung
Miss Katherine Murray

Raleigh, N. C.
Mrs. Helen Ferguson
Mrs. Meta Godwin
Miss Jean Ward
Miss Agnes Wilson
(co-ordinator)

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Miss Grace Carter
Miss Alice Champion
Mrs. Louise Knott
(co-ordinator)
Mrs. Martha Pleasants.

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W. A. B.

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LEARNING AS REORGANIZATION

CHAPTER I

THE PROBLEM AND ITS BACKGROUND

No one deliberately sets out to make arithmetic, or any other school subject, difficult for pupil learners. On the contrary, teachers and research workers alike strive to make the learning simpler and easier in every way possible. Nevertheless, in spite of the common purpose, widely dissimilar practices and procedures are recommended by different groups, and the same practice or procedure is enthusiastically championed by one group and as enthusiastically condemned by another.

Disagreements of the kind just mentioned are usually the result of either or both of two causes: (1) absence of adequate relevant research data and (2) reliance upon deduction from some theory with regard to the learning process. Obviously, if research data, both relevant and adequate, were available to all parties to the dispute, there would be no dispute. But, lacking relevant data, or possessing but a fraction of the needed data, the disputants tend to base their arguments upon their particular conceptions of the learning process.

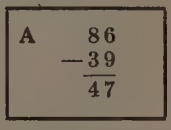
In the study reported in this monograph a typical center of disagreement has been isolated for investigation. That this center of disagreement lies in the field of arithmetic is more or less beside the point. The experimental situation here studied, as it happens, especially favors the investigation of a certain instructional device in third-grade arithmetic. The value of this device has surely not been known because of the absence of appropriate data; nevertheless, if it were to be assessed, its value would be rated as low because of the view of learning which has been held rather generally during the last two decades. Other subject-matter fields would certainly have yielded other instructional devices susceptible to quantitative research; but the arithmetic device was ready at hand and seemed to promise much for the kind of study proposed.

All of this is but another way of saying that the significance of the data to be reported is by no means limited to the particular arithmetic device under investigation, or even to the particular field of arithmetic as such. It is true that the data afford a test of a certain arithmetic device, and so may contribute to improved arithmetic instruction. But the data, it is thought, should do more than

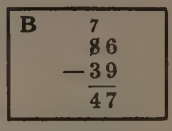
this: they should add to our understanding of the nature of learning. And it is this hope, and not alone the hope of possible changes in arithmetic instruction, which leads to the publication of the extended comparisons and tabulations in this monograph.

THE PROBLEM

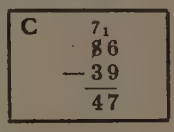
The arithmetic device which lies at the center of this inquiry relates to the teaching of borrowing in subtraction. Example A at the right may be used for illustration. In order to subtract 9 from 6, the 6 of the minuend must first be changed to 16. The child may simply be told to *think* the change, thus: "I can't take 9 from 6, so I borrow 10 from 8, leaving 7; now, 9 from 16 is 7, and 3 from 7 is 4."



It will be observed that by this method (which we may call Method A) the child performs all the operations "in his head," and the form of the example is altered in no way to provide a clue as to what is to be done or a record of what has been done. Some few authorities, and many more teachers, have adopted means which they believe help the child to understand what he is to do. According to this method (Method B), the figure 8 in the minuend is crossed out and a small 7 is written above it, as in Example B.



Now, it is possible that at first even Method B does not give the pupil a sufficiently complete visible record of the process. It does show the change in the tens' figure (from 8 to 7), but it does not show the change in the ones' figure (from 6 to 16). This latter change can be shown by writing a small 1 before the 6 of the minuend, as in Example C. To use Method C, then, the pupil alters the example in two ways before computing at all, except to see that borrowing is required.



Common practice in textbooks clearly favors Method A. Of nine series of texts published since 1935, all except two show this method exclusively, and none show Method C. Only one text shows Method B. The ninth text, in order to rationalize the process, illustrates the borrowing somewhat after the manner of Method C, but does not recommend that pupils alter the example itself in any way.

Thus, Method A is the approved method, and Method C has no advocate at all among textbook writers. Indeed, its use in a text-

book would immediately arouse a storm of criticism. The mere fact that we have (or have had) no data whatever on the comparative merits of the three methods mentioned above would not prevent positive assertions that Method C is totally indefensible. The argument would be that the method is slow, cumbersome, apt to be confusing, uneconomical, and almost certain to be retained permanently as a "crutch." By contrast it is held that from the outset children should be exposed only to forms and procedures which make for efficient performance. Method A satisfies these specifications; Method C does not; hence, Method A is the one to teach. Here then is a good case of reliance upon a particular conception of the learning process in the absence of convincing data. Method C would be rejected and Method A accepted because of deductions from a prevalent learning theory.

The particular theory which supports this decision is clearly revealed in the arguments cited against use of crutches in general. According to this theory, learning should proceed smoothly and uninterruptedly from initial status to the desired outcome. The adult, or the expert in subtraction, does not, for example, alter examples when he must borrow. He performs the whole operation "in his head" because writing out the crutch (1) is time consuming and therefore uneconomical, (2) yields no greater accuracy, and (3) makes the paper look "messy." The argument proceeds: since the adult does not use the crutch, the child should not be shown the crutch. Instead, the child should, as nearly as he can, do exactly as the adult does. If the child is taught the crutch, he is taken off the main highway of learning certainly to no good end, and probably to his detriment. At best he will eventually have to "unlearn" the crutch which he has so laboriously mastered, and the "unlearning" may be so arduous as never actually to be accomplished. In this case the child remains forever handicapped in subtraction.

This is one side of the debate. There is another side, though this side is but seldom presented. Those who might be inclined to defend Method C would rest their case on a different view of the learning process. They would not be so eager to secure economy at first and would not be disturbed by excursions from the main highway of learning, provided that these excursions result in gains in meaning. Furthermore, they would minimize the danger that the crutch once learned might become a permanent habit and would expect the later transition to more economical procedures (which make for greater efficiency) to be both easy and natural.

SIGNIFICANCE OF THE PROBLEM

Thus the issue is joined, and it is seen fundamentally to revolve about different conceptions of the course of learning. To the one group of theorists, learning experiences take their pattern from the final product, and practice on this pattern is initiated at once. To the other group, the nature of early learning activities need not resemble the final activities at all; the measure of their worth is not likeness with adult performance, but contribution to understanding.

These differences in learning theory, it will be recognized, are by no means confined to discussions about arithmetic; on the contrary, they are encountered rather generally in the literature of education. The present study, then, presents, as it were, a test case of the validity of the two theories. Whether the arithmetic crutch whose use is investigated proves to be a helpful or harmful instructional device is incidental to the larger problem, namely, the course of learning.

OUTLINE OF THE MONOGRAPH

After the procedure employed in this investigation has been described in Chapter II, the collected data are presented in Chapters III and IV. In these two chapters little attention is given to the significance of the findings for psychological theory. Instead, the discussion relates almost wholly to the crutch purely as an instructional device in teaching how to borrow in subtraction. The reader of these chapters, however, will do well, as he reads, to note facts of importance for the learning process in general, and not merely for learning in arithmetic.

In Chapter III data are reported on one of the two major aspects of the problem as related to arithmetic, namely, What effect has use of the crutch (Method C, p. 4) upon success in learning to subtract with borrowing? "Success" is here defined in the usual terms of rate and accuracy of work.

In Chapter IV the second major aspect of the arithmetic problem is considered: What actually happens once the crutch has been learned? Does it become a fixed habit, or is it discarded with more or less ease? If the latter, under what conditions, and by what type of child? The facts presented naturally have bearing on the common psychological objections to crutches as influences which retard sound growth.

Chapter V brings together the facts reported in Chapters III and IV and considers their significance both for the teaching of

arithmetic and for learning theory. It is suggested that the reader who is less interested in the details of the experimental procedure than he is in the educational and psychological implications of the results may wish to turn at once to Chapter V.

A rather extended Appendix contains sample tests and sample directions to the teachers and is interesting chiefly to the student of the experimental procedure employed in this study.

CHAPTER II

THE EXPERIMENTAL PROCEDURE

Details as to the experimental procedure are discussed in this chapter under the following headings:

1. Place and Time (p. 8)
2. Learning Materials (p. 8)
3. Subjects (p. 10)
4. Experimental Sections (p. 11)
5. Directions to Teachers (p. 16)
6. Measures Obtained (p. 18)
7. Chronology of the Experiment (p. 22)

1. PLACE AND TIME

The investigation involved 590 third-grade pupils in the schools of Charlotte, Greensboro, Raleigh, and Winston-Salem, North Carolina. In each city four different third-grade classes from widely separated schools were used. The selection of classes at some distance from each other made it possible to maintain and control differences planned in the investigation.[1]

The experiment proper covered about two months of school time. All classes within a given city began, continued, and concluded the experiment according to a time schedule agreed upon by the four co-operating teachers. As between cities, the time schedules differed by less than two weeks; all classes began experimental teaching between November 10 and November 22, 1937.

The dates given refer to the "experiment proper," that is, to the experimental period as originally planned. As a matter of fact, late in the study it became evident that additional tests should be given to all pupils after the cessation of instruction in borrowing. One of these tests was administered in all cities two weeks after the experiment proper had closed, and the other about the middle of May.

2. LEARNING MATERIALS

The textbook adopted by the state of North Carolina and used in all the experimental cities except one was a special state edition

[1] The school systems will be designated by numbers only, thus: City I, City II, City III, and City IV, so as to prevent identification. The order of the Roman numerals does not correspond to the alphabetical order in which the cities are mentioned above.

of *The New Day Arithmetics*, by Fletcher Durell, Harry O. Gillet, and Thomas J. Durell, and published by the Chas. E. Merrill Co. (1931). The teachers of the one group of schools which had not been using the state text agreed to do so for the purposes of the experiment and in the middle of October altered their teaching so that their pupils would be ready a month later to study the matter in the state text.

This edition of *The New Day Arithmetics* is organized according to a "spiral" plan, with large "loops." Chapter I of the third-grade text reviews the simple combinations in addition and subtraction. Chapter II is devoted exclusively to addition, and Chapter III exclusively to subtraction. This arrangement lent itself admirably to the proposed study by insuring exact control over the subtraction content to be taught to the children.

The first three pages of Chapter III (pp. 79-81) consist in a review of easy subtraction with two- and three-place numbers without borrowing. These pages afforded an excellent opportunity for each teacher to reteach the simpler aspects of subtraction and thus to guarantee that their pupils would be ready for the essentially new feature (borrowing) with which the experimental study was concerned.

Pages 82-100 contain the matter taught to all pupils in all sections during the experimental period. Briefly, the contents of these pages are as follows:

Page 82: first presentation of borrowing; examples of the type $65 - 28$; the explanation followed by twenty-one practice exercises. The process of borrowing taught is that of subtractive decomposition.

Page 83: presentation of borrowing by the method of subtractive equal-additions. Page omitted in this study.

Page 84: presentation of borrowing by the method of additive equal-additions. Page omitted in this study.

Page 85: subtraction of the types $474 - 226$ and $965 - 49$; twenty-five practice examples.

Page 86: subtraction with 0's, of the types $60 - 7$, $346 - 239$, and $435 - 428$, with twenty-five practice examples. Of these, twenty were worked by the pupils and sent to Duke University for study; this page constitutes "Practice Page I"; see page 20, following.

Page 87: subtraction with money, of the types $\$5.25 - \$2.18$, and $\$1.25 - \$1.16$; twenty practice examples.

Pages 88-90: verbal problems involving subtraction with borrowing.

Page 91: upper half, four verbal problems; lower half, twenty practice examples in subtraction, mixed types.

Pages 92-95: problems and practice examples in subtraction with borrowing; bottom half of page 95, thirty-six examples, the first eighteen of which were worked by pupils and sent to Duke University as "Practice Page II."

Pages 96-100: verbal problems, diagnostic test in subtraction, and practice examples.

The process for borrowing in subtraction was taught on page 82, as shown below. The method will be recognized as that of subtractive decomposition, the only method which was shown to any of the children in the experiment. The textbook method will also be recognized as what has been designated as Method A on page 4, preceding. The following matter is quoted from the textbook:

LEARNING TO CARRY IN SUBTRACTION

Betty wants a doll that is in Mr. Bell's store window. It costs 28 cents. Betty has 65 cents. If she buys the doll, how much money will she have left?

You must subtract to find out.

$\begin{array}{r} 65 \\ 28 \\ \hline 37 \end{array}$	5 − 8. But you cannot take 8 cents from 5 cents. So change 5 cents to 15 cents by using 1 of the 6 dimes. 15 − 8 = 7. Remember you have only 5 dimes left. 5 − 2 = 3. She will have 37 cents left. Check the example.

3. SUBJECTS

Table 1 presents the data on the subjects used in the various parts of the experiment. Column 2 shows the enrollment (a) in the sixteen classes by cities and (b), at the bottom, by experimental sections (explained in Section *4* below).

In the study of relative success in learning to borrow, which is the first of the two major problems to be considered, eliminations were made as follows: (a) all pupils with IQ's of 74 and below, (b) all pupils whose test records were too incomplete to justify inclusion for purposes of evaluation, (c) all repeaters with low IQ's (even if their IQ's surpassed 74), and (d) individuals whose measures of general or arithmetical ability could not be matched in making up the experimental sections. The result of these eliminations was a reduction in the number of subjects from 590 to 419, as shown in Column 3 of Table 1.

TABLE 1
DISTRIBUTION OF SUBJECTS

	Total Enrollment	Efficiency Study (Equivalent Groups)	Crutch Study
(1)	(2)	(3)	(4)
City I			
Section NC†	32	27	..*
Section R †	32	21	25
Section D †	31	28	31
Section O †	32	26	28
Total	127	102	84
City II			
Section NC	47	24	..*
Section R	29	24	28
Section D	40	27	29
Section O	42	30	35
Total	158	105	92
City III			
Section NC	37	28	..*
Section R	46	26	33
Section D	44	32	38
Section O	40	26	26
Total	167	112	97
City IV			
Section NC	27	22	..*
Section R	42	29	31
Section D	30	20	23
Section O	39	29	31
Total	138	100	85
Grand Totals	590	419	358
Totals by Sections			
NC	143	101	...*
R	149	100	117
D	145	107	121
O	153	111	120

*The pupils of the NC Sections were not used in the study of what happened to the crutch, since they were not taught the crutch.
†Section NC=children who were not taught the crutch at all (non-crutch).
Section R =children who were *required* to use the crutch throughout the experiment.
Section D =children who were taught, then *denied*, the crutch.
Section O =children to whom use of the crutch was *optional* after a certain time.

The second major problem relates to what may be called the history of the crutch; that is to say, an account of what happens to the crutch once it is learned. For this study the pupils in Section NC were not used, for they were not taught the crutch. All other pupils were retained whose records were reasonably complete.

4. EXPERIMENTAL SECTIONS

The four classes in one city were organized in exactly the same way as were the four classes in each of the other three cities. In

this way the investigation actually comprised four separate but parallel studies which, it was thought, might be useful in checking the dependability of the findings.

In each city one class was to be taught to borrow in subtraction precisely according to the textbook plan; in other words, without a crutch. This section will be referred to as Section NC, the non-crutch section. In the other three classes the crutch was to be taught, but under different conditions. In Section R (the second of the four classes) the crutch was to be taught and was to be *required* (hence, Section R) in all assigned work throughout the experiment proper. In Sections D and O the crutch was to be taught, and its use was to be required for a limited time, after which the pupils of Section D were *denied* the crutch, and those in Section O were allowed to use it or to use the shorter textbook method as they wished (D = denied, and O = optional).

The initial equivalence of the four classes was therefore a matter of some concern. Four steps were taken to insure this equivalence. (a) The elementary supervisor or the superintendent in each city was asked to select from the total number of third-grade classes in his system four classes which drew from the same general economic and cultural areas. (b) In making the selection the supervisor or the superintendent was asked to bear in mind one other consideration: only teachers should be chosen who could and would co-operate willingly and intelligently in the proposed experiment. (c) An interview was held with the selected teachers in each separate city, in part to make sure of their co-operativeness, interest, and ability, and in part to assign them to the experimental sections. (d) Tests were given to all pupils at the start of the experiment—tests of general ability and of arithmetical ability. Scores on these tests were used at the conclusion of the study to make up composite sections (NC, R, D, and O) which would be as nearly equal as possible in these characteristics.

That these precautions met with some success (though not with as much as was anticipated) is evidenced by certain facts. (a) and (b) City I had thirty-three third-grade classes from which four were to be chosen; correspondingly, City II had eighteen classes, City III had fifteen, and City IV had twenty-five. From numbers such as these there was reason to believe that comparable classes and equally competent teachers might well be chosen. (c) So far as the teachers were concerned, the interviews afforded an opportunity to form a judgment. In each city all four teachers were

eager to participate in the study, and in every instance they could be assigned to the particular section which was to be taught in the manner which they preferred.

(d) The measures obtained from the intelligence test and from the arithmetic test revealed, however, that in spite of the steps taken and described in (a), (b), and (c), the four sections within each city were not comparable. On this account it became necessary to abandon the plan of presenting the data separately by cities, thus reporting four separate experiments, and to combine the pupils from corresponding sections in the four cities to secure comparable groups. The close similarity of the experimental procedure in the different cities seemed to justify this procedure.

The data in Tables 2 to 5 show the degree to which the four composite sections were actually equivalent. Table 2 contains the data on IQ's, derived in Cities I, III, and IV from the Kuhlmann-Anderson Test, Fourth Edition, and in City II from the Otis Primary Intelligence Examination. All tests were given and scored by

TABLE 2

DISTRIBUTION OF IQ'S, EXPERIMENTAL SECTIONS

IQ	DISTRIBUTION BY SECTIONS			
	NC	R	D	O
(1)	(2)	(3)	(4)	(5)
122	7	1	5	1
118	2	3	13	7
114	12	11	4	16
110	17	11	16	18
106	20	22	10	18
102	9	13	18	15
98	12	16	8	4
94	4	10	9	12
90	5	7	12	7
86	7	1	8	10
82	3	3	2	1
78	3	2	0	2
74	0	0	2	0
Totals	101	100	107	111
Means	103.5	102.2	102.3	103.0
σdist	10.9	9.0	11.4	10.3
σM	1.1	0.9	1.1	1.0

The Critical Ratios, obtained here as in all other places, by use of the formula are as follows:

NC — R = 0.92 NC — D = 0.77

NC — D = 0.34 R — D = 0.70

R — O = 0.59 D — O = 0.47

$$\text{Diff. } \frac{M_1 - M_2}{\sqrt{\frac{\sigma 1^2}{N_1} - \frac{\sigma 2^2}{N_2}}}$$

Duke University graduate students who were thoroughly competent for the task. The largest difference between mean IQ's is for Sections NC and D: 103.5 – 102.2, or 1.3 IQ points. This difference is not a reliable one, nor is anyone of the other differences between sections (see the footnote to Table 2). Moreover, this difference, though small, fortunately favors the section which in this investigation is the "control" group, thus giving advantage, if any, to the teaching method which represents common or typical practice.

Table 3 is constructed in a manner similar to that for Table 2. The scale values, of course, represent years and months of mental age, for which the data on CA were obtained directly from the school records. The largest difference between section medians is 1.9 months of mental age (Section D-Section R). Neither this difference nor any of the other differences is statistically reliable.

Tables 4 and 5 are based upon scores made on a specially constructed arithmetic test. This test, reproduced on page ii of the

TABLE 3

DISTRIBUTION OF M.A.'S, EXPERIMENTAL SECTIONS

MA	DISTRIBUTION BY SECTIONS			
	NC	R	D	O
(1)	(2)	(3)	(4)	(5)
10-6	4	4	3	3
10-3	4	2	6	4
10-0	7	3	4	7
9-9	14	10	13	12
9-6	11	11	19	19
9-3	12	23	15	22
9-0	9	13	20	12
8-9	13	13	15	14
8-6	6	12	7	5
8-3	7	3	3	5
8-0	5	0	1	4
7-9	3	4	1	3
7-6	3	0	0	0
7-3	0	1	0	0
7-0	3	1	0	1
Totals	101	100	107	111
Means	9-1.8	9-1.2	9-3.1	9-2.3
σdist	9.84 (mos.)	8.4	6.7	7.8
σM	1.0	0.8	0.6	0.7
Critical Ratios:	NC — R = 0.47 R — D = 1.80	NC — D = 1.11 R — O = 0.98		NC — O = 0.49 D — O = 0.81

Appendix, consisted of twenty examples, half in addition and half in subtraction, and all well within the ability of the children. None of the subtraction examples involved borrowing, and the addition examples were also of the kind taught in Chapter II of the textbook. Incidentally it may be said that the reliability of the test is represented by $r_{11} = .70$.[2]

This test yielded two measures: a measure of accuracy (number of correct answers) and a measure of rate. The rate measures were secured by having all pupils encircle the example upon which they were working when a signal was given by the teacher at the end of seven minutes. After marking this example, the pupils continued with the test until all except 10-15 per cent of the pupils had completed it. The directions were to work as rapidly as possible without endangering accuracy.

Both the figures for accuracy (Table 4) and those for rate (Table 5) reveal the skewed distributions. The average accuracy median was 17 out of a possible 20, and the average rate median, 20 out of 21. This skewness to the left is precisely the condition which was desired, for it shows that the pupils had fairly well mastered the computational skills which were essential to learning how to borrow in subtraction.

One of the Critical Ratios for accuracy is very nearly reliable, that for Sections R and O (2.9), and another, for Sections NC and R, is more than 2.0 (2.2). Still, the latter is much too small to represent certainty, and the former relates to two of the "experimental" sections (that is, does not include the "control" or non-crutch section), and may be disregarded for the time being. One point should be made in passing, namely, that the children of Section O scored lowest of the four sections. This deficiency, which is here not a statistically reliable one, will be evident throughout the experimental report.

The rate measures in Table 5 are interpreted in terms of medians rather than means. This change in statistical treatment is made necessary by the large undistributed samples in the highest interval –21. All pupils who completed the test of twenty examples before the time signal was given are credited with rate scores of 21. The lack of discrimination between the scores in this interval does not affect the median as it would the mean. It does affect the com-

[2] All reliability coefficients were obtained from samples of one hundred test papers, by correlating scores on test-halves and stepping up the resulting coefficient by the Spearman-Brown formula. The low reliability of the Pretest is accounted for by the skewness and short range of the distribution.

TABLE 4

RELATIVE ACCURACY OF THE FOUR EXPERIMENTAL SECTIONS ON THE PRETEST

Scores	ACCURACY, BY SECTIONS			
	NC	R	D	O
(1)	(2)	(3)	(4)	(5)
20	15	24	17	12
19	23	21	26	24
18	8	10	11	12
17	16	18	12	12
16	5	7	12	11
15	8	4	9	12
14	6	7	4	5
13	5	1	6	4
12	4	4	1	4
11	3	0	4	4
10	1	1	2	3
9	3	2	1	4
8	2	0	0	0
7	1	0	1	2
6	1	0	0	1
5	0	1	1	1
Totals	101	100	107	111
Means	16.3	17.3	16.8	16.0
σdist	3.4	2.9	3.1	3.7
σM	0.3	0.3	0.3	0.4

Critical Ratios: NC — R = 2.2; NC — D = 1.1; NC — O = 0.6; R — D = 1.2; R — O = 2.9; D — O = 1.7

The distributions in this table and those in following tables are notably skewed to the left. Since this condition is uniform for all sections, it is believed that it does not affect the validity of the conclusions.

putation of Q_3, and so of Q, but in the case of Table 5, this effect is unimportant: all sections had the same median rate of work, namely, 20 examples.

In summary, then, it seems fair to say that the four composite sections may be regarded as comparable at the start of the investigation, whether the basis of comparison be intelligence measures or arithmetical measures. The only exception to this statement is that the pupils of Section O appear to have been slightly inferior in arithmetical ability.

5. DIRECTIONS TO THE TEACHERS

At the interviews already referred to, the purpose and the general plan of the investigation were thoroughly explained. A "coordinator" was selected for each group, to be responsible for maintaining uniformity among the schools of her city and to serve as a

TABLE 5

RELATIVE RATE OF WORK OF THE FOUR EXPERIMENTAL SECTIONS ON THE PRETEST

Scores	RATE, BY SECTIONS			
	NC	R	D	O
(1)	(2)	(3)	(4)	(5)
*21	30	31	19	52
20	33	25	58	16
19	3	3	5	5
18	6	2	4	5
17	5	7	4	4
16	8	7	5	5
15	0	6	3	5
14	0	3	1	4
13	5	0	2	2
12	3	10	2	5
11	2	3	1	5
10	0	2	2	0
9	1	0	1	2
8	0	0	0	0
7	3	0	0	1
6	0	1	0	0
5	2	0	0	0
Totals	101	100	107	111
Medians	20	20	20	20
Q	2.0	3.0	0.5	2.5
†P. E. Mdns.	0.2	0.4	0.3	0.1

*All pupils who finished the twenty examples of the test before the time signal are arbitrarily assigned scores of 21. The large undistributed number of cases in the 21-interval makes dubious the calculation of Q_3, and hence of Q, in the cases of Sections NC, R, and O. However, the exact equivalence of the medians in itself virtually establishes the comparability of the four sections as to rate.

†P. E. Mdns. is computed according to the formula:

$$\text{P. E. Mdn.} = \frac{5}{4}\sqrt{\frac{Q}{N}}$$

(see Charles C. Peters and Walter R. Van Voorhis, *Statistical Procedures and Their Mathematical Bases*, p. 125).

general clearing house for ideas and correspondence. Arrangements were made for meetings as needed, in order to decide upon dates in the time schedule, to determine what, if any, supplementary practice should be used, and so on.

Then each teacher was supplied with a sheet of "General Instructions" (Appendix, p. iii), which gave her a permanent record of (a) the problem, (b) the experimental schedule, (c) the types of record to be kept (see Section *6*, below), (d) the subtraction method to be taught, (e) directions as to home work (none), and so on.

Next, a copy of "Test Instructions" (Appendix, p. v) was given

each teacher. The tests will be described, and the method of administering them outlined, in the succeeding section of this report.

Last of all, each teacher was provided with typewritten directions for teaching her particular section (Appendix, pp. vii-xvi), which told her in detail precisely what she was to do in connection with each page of the textbook. At the conference ample time was allowed for the study of these directions and for questions at points of difficulty. So completely satisfactory were these directions that the co-ordinators had to communicate with the director of the experiment but a very few times. Each teacher seemed to understand what was expected of her and to be anxious to start the experimental teaching.

6. MEASURES OBTAINED

As explained on the sheet of "General Instructions," five types of measure were originally contemplated, namely, (a) scores on tests, (b) practice pages, (c) teachers' logs or diaries, (d) interviews and conferences with pupils, and (e) personal data on pupils. Each of these is described below.

a. Tests

Five arithmetic tests were used. One of these, which hereinafter will be called the Pretest, has already been described. Its purpose was (1) to make sure that the pupils in all sections were "ready" for borrowing in subtraction and (2) to balance the experimental sections for the comparisons to be made. The data on this test have already been presented (Tables 4 and 5).

Perhaps a word of explanation is in order with respect to the method of administering the Pretest (and all other tests). Greater differences in scores for accuracy (that is, number of correct answers) could have been obtained if a short time limit had been enforced, thus making it impossible for more than a very few individuals to finish. The differences thus obtained would, however, have been somewhat artificial, and, moreover, the accuracy measures themselves would probably not have been valid. Included in these accuracy measures would have been the effect of time pressure, which is a variable factor for different children. It seemed much wiser, therefore, to secure the accuracy measures and the rate measures under test conditions which allowed the children to work more normally. The procedure employed for this purpose has already been described: the pupils encircled the example they

were solving when a time signal was given, and then went on to finish the test.

Besides the Pretest, two other tests were originally planned, one to be given in the middle of the experiment, after important changes in instruction had been made in teaching Sections R, D, and O; the other at the conclusion of the instructional period, that is, after Chapter III of the textbook had been taught. These two tests (Tests I and II) were prepared and given in a manner similar to that for the Pretest. Later, two other tests (Tests III and IV) were added to the measurement program. Test III was given in all classes two weeks after the end of instruction on borrowing in order to ascertain the effect of lapse of time and of interpolated instruction in kinds of arithmetic other than subtraction. (Chapter IV of the textbook takes up multiplication.) Test IV was given to Sections NC, D, and O early in the middle of May, to study the use of the crutch yet again, this time after a lapse of four months from Test III.

Test I was given in all the schools about three weeks after the Pretest, which is to say, after Section NC had learned how to borrow in the manner described on page 10 preceding, and after Sections R, D, and O had learned to borrow by using the crutch. The three crutch sections, it will be recalled, were not only taught the crutch, but were required to use it in all their assigned work. On taking Test I (and later, Test II) they were, however, given no instructions whatsoever as to whether they should use the crutch on the subtraction examples, and the teachers were directed to parry any questions on this point.

Test I consisted of twenty examples, fifteen in subtraction and five in addition (to keep the children "on their toes"). Of the subtraction examples, ten required borrowing and five did not. A copy of the test ($r_{11} = .88$) will be found on page xvii of the Appendix.

Test II was administered after all the work, new matter, reviews, problems, diagnostic tests, etc., in Chapter III had been presented. Like Test I, it comprised twenty examples, fifteen in subtraction and ten of these requiring borrowing. A copy of the test (reliability, $r_{11} = .91$) will be found on page xviii of the Appendix. The test was given exactly as was Test I.

Test III consisted of thirty-six examples, all of them requiring borrowing. This test was divided into three parts by heavy lines, and these parts were administered differently. When the first part (twelve examples) was given, nothing at all was said about the

crutch in any section, and the time signal was announced after three minutes of work. When the second part was given, the teacher announced that the crutch should *not* be used and that penalties would be applied if the crutch appeared on the papers. The time signal was again given after three minutes. All the examples up to this point were alike and of a familiar type: they called for borrowing from the tens' figure only. The twelve examples in the third part, however, were new, the first six requiring borrowing from hundreds only, and the second six, from both hundreds and tens. The purpose of the last part of the test was to find out how well the children of the various sections could "transfer" their skills to new kinds of subtraction, and the purpose of the first part, to ascertain how well they had retained their ability to subtract after an interval of some weeks of direct instruction thereon.

A copy of Test III appears on page xix of the Appendix. (The reliability coefficient for Part 1 is very low because of the extreme skewness in the actual scores. The distribution shows a particularly large skewness to the left.) The reliability coefficients are: .54, .81, and .88.

Test IV, a copy of which will be found on page xx of the Appendix, consisted of twenty-four examples, all in subtraction. The first three examples in each of the six rows were precisely of the kind taught during the period of experimental instruction. The last column of six examples represented types taught after the conclusion of the experiment. Test IV was administered to Sections R, D, and O exactly as were Tests I and II, with a time limit, but with no mention of the crutch to any section. Its purpose was entirely to study the continuance or discontinuance of the use of the crutch; hence, no data will be reported from this test on the comparative rate and accuracy of subtraction for the three crutch sections.

b. Practice pages

To supplement the measures from the tests, two other measures of learning were obtained by having pages of assigned work collected from the pupils. These practice pages, referred to on pages 9 and 10 preceding, were arranged to come (1) shortly after the three crutch sections (R, D, and O) had all alike been taught the crutch and required to use it (in connection with p. 86 of the textbook) and (2) after important changes had been made in the instruction given to the crutch sections (in connection with p. 95 of

the textbook). While practice pages were collected from the various NC-sections as well as from the crutch sections, no data for the former are reported for them for the reason made clear in the following paragraphs.

The practice pages supplied no certain knowledge concerning the relative success of the sections in learning to borrow, and hence are not cited in Chapter III where that problem is considered. After all, the examples on the two textbook pages were assigned to provide learning practice, and the control of conditions in order to secure rate measures, accuracy measures, or both, would probably have interfered seriously with the learning which was contemplated.

The data from the practice pages are, however, of value for the second major problem of the study, and so they are included and considered in Chapter IV. Examination of the work pages revealed evidence (1) on the degree to which the children who had been taught the crutch actually used it in preparing assigned lessons (Practice Page I), and (2) on the degree to which the crutch children discarded the device under varying conditions of instruction (Practice Page II), to be described in Section 7 of this chapter. As will be noted in the special instructions given the teachers of all sections (Appendix, pp. iii and iv) *nothing at all* was said about the crutch when these pages of examples were assigned—children might use it or not. This plan was adopted, obviously, in order to see what the crutch children would do voluntarily, whether they would use the device consistently, use it irregularly, or try to get along without it entirely.

c. Teachers' logs or diaries

All teachers were asked to keep complete records of classroom happenings which might be significant for the experiment, such as questions asked by pupils, evident difficulties, objections raised (as to the continued use of the crutch), and so on.

Most of the sixteen teachers kept exhaustive diaries, and the rest, except for five, kept some kind of diary. Of these five exceptions, four were teachers of NC-sections, and they stated that "nothing unusual" occurred to make it advisable to keep records. The fifth teacher taught an O section. Data from the teachers' logs will be cited as needed in Chapters III and IV.

d. Interviews and conferences with pupils

When the plan of the investigation was first outlined, it seemed probable that occasional classroom visits would be necessary to main-

tain control of the teaching methods. These proved not to be needed, for the original directions, supplemented by a few letters and telephone conversations, were adequate to insure that the children were being taught as they were supposed to be taught. Evidence on this point was continuously available in the sets of papers, seven in all, which were sent to the investigation headquarters.

Personal interviews with pupils at various stages of learning would unquestionably have yielded data not obtained from the test papers and the practice pages. However, practical difficulties of finance and of scheduling visits made the visits impracticable. Still, in spite of the absence of interview data, the report of the experiment will, it is thought, be accepted as unusually complete.

e. Personal data concerning pupils

The only kinds of personal data which were collected have already been mentioned: data on age, on absences, and on normal or slow progress through the grades.

7. CHRONOLOGY OF THE EXPERIMENT

The chart (p. 23) provides a convenient method of summarizing the various aspects of the experimental instruction and of showing the schedule of the investigation. The charts for the other three cities do not differ materially from this chart for City III, except in the matter of dates. All sections within a given city, as has already been said, held to the same schedule.

All four sections, it will be observed, reviewed pages 79-81 of the textbook alike, and so were gotten ready to learn to borrow in subtraction.

Beginning with item 3 of the list in Column 1, Section NC split away from Sections R, D, and O. Section NC continued to follow the textbook; its members did not learn the crutch. Sections R, D, and O, by contrast, were taught the crutch and were required to use it in all their regularly assigned work. Thus, these three crutch sections were alike in their experience with the device, and as a group unlike Section NC, throughout pages 82-90 of the textbook. Practice Page I, collected at the middle of this period, and Test I, given at the end of the period, provide evidence on the extent to which the crutch was used when no specific demand was made for its use; and Test I provides evidence on the relative success of the four sections in learning to borrow in subtraction.

Beginning with item 7 of the chart, marked changes were introduced in the training of the crutch sections. (Section NC contin-

CHRONOLOGY OF THE EXPERIMENT, CITY III

	Section NC Miss_____	Section R Miss_____	Section D Miss_____	Section O Miss_____
1. Text 79-81 Nov. 10	Teach thoroughly	Same as NC	Same as NC	Same as NC
2. Pretest Nov. 19	Give as directed	Same as NC	Same as NC	Same as NC
3. Text 82-85 Nov. 22	Teach exactly per text	Teach crutch and require on all work	Same as R	Same as R
4. Practice Page I (Text, p. 86) Dec. 7	Special instructions	Same as NC	Same as NC	Same as NC
5. Text 87-90 Dec. 8	Teach exactly per text	Teach crutch and require on all work	Same as R	Same as R
6. Test I Dec. 17	Give as directed	Same as NC	Same as NC	Same as NC
7. Text 91-95½ Dec. 18	Teach exactly per text	Teach crutch and require on all work	Show short method and urge abandonment of crutch	Show short method and make it optional. Do not urge or forbid use of crutch
8. Practice Page II (Text, p. 95½) Jan. 7	Special instructions	Same as NC	Same as NC	Same as NC
9. Text 96-100 Jan. 8	Teach exactly per text	Continue to require crutch	Continue urging abandonment of crutch	Say nothing at all about crutch
10. Test II Jan. 14	Give as directed	Same as NC	Same as NC	Same as NC
11. Test III Jan. 28	Give as directed	Same as NC	Same as NC	Same as NC
12. Test IV May 12	Give as directed	Same as NC	Same as NC	Same as NC

ued as before to do all work without knowledge of the crutch.) Now for the first time the children of Sections D and O were shown how to borrow without the device. That is, they were shown how to perform all the operations mentally and without altering the example before computation. While the pupils in Section R remained ignorant of the shorter method and were still required to use the crutch, the children of Section D were urged to abandon the device in favor of the shorter form, and those of Section O were allowed

to adopt the shorter form if it commended itself to them. The variation of instruction which was introduced beginning with page 91 and which was maintained throughout the rest of the experiment up to Test II, was intended to isolate some of the instructional factors which might be related to continued use of the crutch. Data from Section R were supposed to furnish some light on the question, Will children continue to use the crutch, even if they do not need it, merely because they are told to use it; or, will they tend to abandon it in spite of teachers' directions to the contrary? Data from Section D were supposed to help answer the question, How much urging on the part of teachers is necessary in order to get children to surrender a crutch which previously they have been required to use regularly? Similarly, data from Section O, it seemed, should be valuable in connection with the question, Will children who have learned a crutch drop the cumbersome procedure in favor of the more direct procedure when they are merely made aware of the possibility of the latter?

What may be called the third phase of the experiment (the first being a review of subtraction without borrowing, and the second, the period of instruction on the crutch) lasted through a period of approximately two weeks, when Chapter III of the textbook was completed. During this period one practice page (textbook page 95) and one test (Test II) were arranged in order to secure information both on the use of the crutch and on success in learning to subtract.

The third phase of the experiment was concluded about the middle of January. By this time all work on subtraction had been completed, and the children then began the study of multiplication. About two weeks later, Test III was given, again to check the use of the crutch and relative efficiency in subtraction. During the period elapsing between Test II and Test III nothing had been said in any of the classrooms about the crutch. As a matter of fact, in the interim, abstract practice in computation had dealt little with subtraction. Measures obtained from Test III, therefore, were expected to show the effect of a short interval of time on the factors under study.

Test IV, given about May 10, and therefore some four months after Test III, provided evidence of the effect of a still longer period of time upon use of the crutch. (The data from this test are restricted to a survey of the use of the crutch alone.)

Figure 1, which follows, shows diagrammatically the time relations between the different features of the experiment.

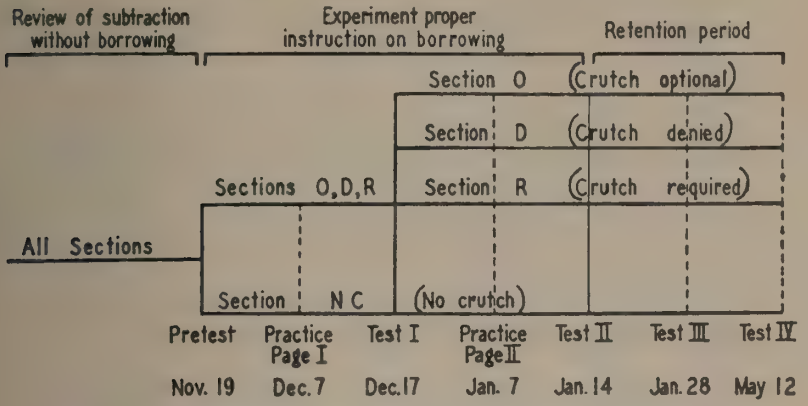


FIG. 1. Time schedule and instructional variations by sections.

CHAPTER III

THE EFFECT OF THE CRUTCH ON LEARNING TO BORROW

The first of the two major problems on which data were collected is the effect of the crutch on learning to borrow in subtraction. Stated in question form and in terms of the four experimental sections, the problem becomes, Did the children who were taught the crutch surpass, or equal, or fail to match the non-crutch children in learning to borrow?

Evidence on this problem is of three types; the crutch and the non-crutch children will be compared as to their relative

1. Evidence on accuracy of subtraction (p. 26);
2. Evidence on rate of work of subtraction (p. 35);
3. Understanding of borrowing in subtraction (p. 38).

A section of this chapter will be devoted to each of these types of evidence. In addition, a fourth section (p. 43) will be devoted exclusively to the three crutch sections (taught differently, it will be recalled, after Test I), to ascertain their relative success in learning to borrow.

1. EVIDENCE ON ACCURACY OF SUBTRACTION

a. Gross data

Tables 6 and 7 report the data for the four experimental sections with regard to accuracy of subtraction in Tests I, II, and III. The two tables are similar in construction and are to be read in the same manner; thus, the first row of Table 6 shows that a perfect score of 15 was made by 18 children in Section NC, by 37 in Section R, by 38 in Section D, and by 34 in Section O. Columns 6-9 supply for Test II the corresponding frequencies of this score for the four sections. The data for Test III are found in Table 7, where the scores on the three parts of the test are separately summarized. The important facts in both tables are given in the last four rows.

The four sections started out reasonably equal in arithmetical accuracy (Table 4), Section O being slightly inferior to the other sections. The results from Test I reveal their relative accuracy shortly after they had been taught to borrow, Sections R, D, and O with the crutch, Section NC without it. At this time all three crutch

TABLE 6

DISTRIBUTION OF ACCURACY SCORES ON TESTS I AND II, FOUR EXPERIMENTAL SECTIONS; FIFTEEN SUBTRACTION EXAMPLES

Score	Test I, by Sections				Test II, by Sections			
	NC	R	D	O	NC	R	D	O
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
15	18	37	38	34	26	38	34	21
14	11	16	22	20	23	31	25	21
13	14	16	9	10	17	11	14	18
12	11	8	9	7	6	3	9	12
11	7	6	8	6	4	3	6	7
10	6	4	4	2	3	0	7	7
9	5	1	3	2	4	0	1	3
8	4	2	3	1	0	1	2	2
7	9	0	1	5	0	0	1	5
6	6	0	1	2	2	2	2	3
5	3	2	1	6	4	0	2	6
4	2	2	1	3	3	0	0	2
3	3	2	1	5	3	2	1	1
Total	99	96	101	103	95	91	104	108
Means	10.9	12.9	12.9	12.0	12.3	13.6	12.9	11.8
σDist.	3.4	2.9	2.7	3.8	3.4	2.3	2.6	3.2
σMeans	0.3	0.3	0.3	0.4	0.3	0.2	0.3	0.3

Critical Ratios, Cols. 2-5:
NC — R = 4.7 NC — D = 4.7
NC — O = 2.2 R — D = 0.0
R — O = 1.8 D — O = 1.8

Critical Ratios, Cols. 6-9:
NC — R = 3.6 NC — D = 1.4
NC — O = 1.3 R — D = 1.9
R — O = 5.0 D — O = 2.6

sections excelled Section NC, by average margins of 2.0, 2.0, and 1.1 examples. The differences between Sections NC and R and between Sections NC and D are statistically reliable, but the difference between Sections NC and O is not.

On Test II (Table 6), given immediately at the end of the experimental instruction on borrowing, two crutch sections, R and D, were superior to Section NC (R by a statistically reliable difference), but Section O was inferior to Section NC. In other words, in accuracy, with and without borrowing, Section NC had caught up with and surpassed one of the crutch sections (namely, O) and was approaching another section (D), but was still far from equalling the third crutch section (R).

After the lapse of about two weeks, without instruction on subtraction, Test III was given. On the twelve examples of Part 1, all of a familiar type, Sections NC, R, and D were practically equivalent, and all of them were more accurate than Section O. (The only reliable differences are those between crutch sections.) On the

twelve examples of Part 2 (on which all use of the crutch was denied to Sections R, D, and O), the non-crutch children were somewhat less successful than two of the crutch sections (R and D), but more successful than Section O. (Again the only reliable difference is that between two crutch sections.) In Part 3, in which

TABLE 7

DISTRIBUTION OF ACCURACY SCORES ON TEST III, FOUR EXPERIMENTAL SECTIONS; PARTS 1, 2, AND 3, EACH PART CONSISTING OF TWELVE SUBTRACTION EXAMPLES

Score	Part 1, by Sections				Part 2, by Sections				Part 3, by Sections			
	NC	R	D	O	NC	R	D	O	NC	R	D	O
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
12	47	39	45	22	34	31	36	23	21	13	10	3
11	17	26	21	19	23	24	30	21	10	15	19	5
10	15	11	13	21	12	13	11	11	7	14	12	10
9	5	6	7	14	6	9	8	10	7	12	11	5
8	2	4	6	8	6	5	7	10	8	4	12	6
7	0	2	4	4	4	2	2	6	3	11	11	6
6	1	0	4	5	3	1	3	6	7	4	4	14
5	3	3	0	4	2	2	0	4	4	5	7	13
4	2	1	0	2	2	3	1	2	7	3	3	5
3	0	1	0	1	2	0	2	4	5	4	2	2
2	4	0	1	2	1	3	1	1	2	2	5	7
1	1	1	0	1	2	1	0	5	16	7	5	27
Total	97	94	101	103	97	94	101	103	97	94	101	103
Means	10.3	10.5	10.6	9.4	9.9	10.0	10.3	8.9	7.3	8.1	8.0	5.3
σdist.	2.7	2.1	1.9	2.5	2.7	2.6	2.1	3.1	4.1	3.3	3.3	3.5
σMeans	0.3	0.2	0.2	0.2	0.3	0.3	0.2	0.3	0.4	0.3	0.3	0.3

Critical Ratios:	Part 1	Part 2	Part 3
NC — R =	0.6	0.2	1.6
NC — D =	0.8	1.1	1.4
NC — O =	2.5	2.4	4.0
R — D =	0.4	0.8	0.2
R — O =	4.0	2.6	6.7
D — O =	4.3	3.9	6.4

all twelve examples were of an unfamiliar kind, requiring borrowing from hundreds and from both tens and hundreds, Sections R and D secured more correct answers than did Section NC, but the third crutch section, O, proved to be least accurate of all, its average being less than the other three averages by statistically reliable amounts.

The general picture of the situation may best be shown by means of a diagram or chart. For this purpose the averages in Tables 4, 6, and 7 are converted into per cents by dividing by the number of examples in the test. Thus, the Pretest means of 16.3, 17.3,

16.8, 16.0 (for Sections NC, R, D, and O, respectively) become 82, 87, 84, and 80 per cent when divided by 20, the number of examples. In the case of Tests I and II the divisor is 15, there being that number of subtraction examples in each test; and in the three parts of Test III the divisor is 12. When the scores of the various sections are plotted from these per cents, the relative success of the crutch and of the non-crutch children is clearly revealed. Especially emphatic is the relative loss of Section NC on Test I. The gain on Test II brings the non-crutch children back to their original efficiency, before they were taught to borrow, but leaves them somewhat below the level of Sections R and D, though higher than that of Section O. The relative position of the sections remains about the same on Test III, though the differences are accentuated on Part 3 of this test.

The conclusions from the gross data seem to be these: So far as accuracy in subtraction alone is concerned, the crutch appears to have made important contributions in the early stages of learning to borrow (Test I). Greater experience with the process of borrowing during the next two weeks, however, enabled the non-crutch children to make good their initial loss, and actually to surpass the group of children whose use of the crutch was made optional (Section O, Test II). The non-crutch children continued to gain after instruction on borrowing had been finished, until at the time of Test III they were fully equal to two of the crutch sections and better than the third section in the type of example with which they were familiar (Parts 1 and 2). On the types in which they had to borrow from hundreds and from both hundreds and tens (Part 3), they encountered greater difficulty than did Sections R and D, though less than did Section O.

b. Relative success on the different examples in Test I

As clearly shown in Figure 2, the borrowing crutch made its contribution to accuracy (if it made any at all) in the first stages of learning the process, that is, between the time when the process was introduced and the time when Test I was administered. According to the scores on Test I, none of the crutch sections experienced loss in accuracy of subtraction, and one section (D) may even have improved in accuracy. Such, however, was not the case with Section NC: on Test I the non-crutch children showed considerably less accuracy than they had shown on the Pretest.

In view of these facts it is a matter of some interest to deter-

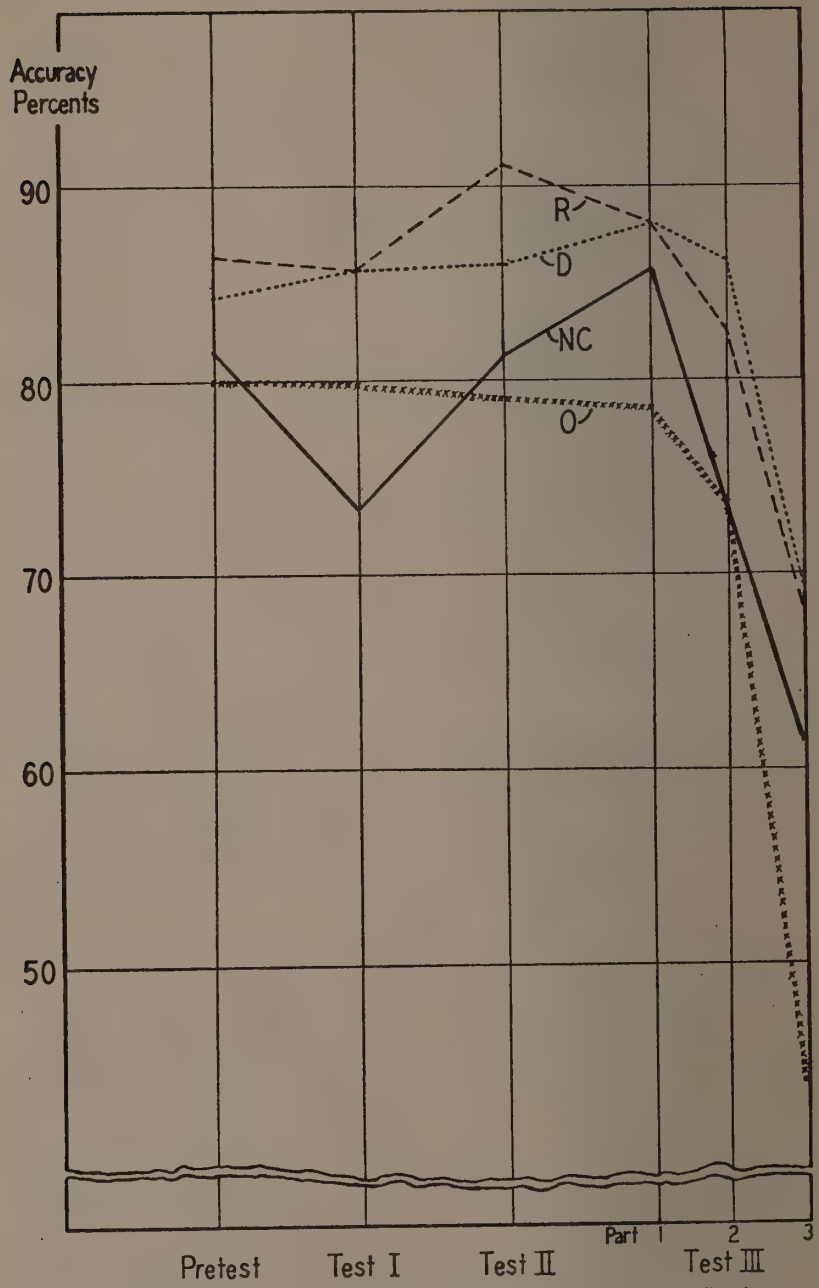


FIG. 2. Percentage accuracy on various tests, Section NC vs. Sections R, D, and O.

mine precisely what happened in Test I. Among other questions were the ones discussed in the paragraphs immediately following: Were the crutch children *generally* superior to the non-crutch children in Test I? Were they superior in the borrowing examples, or in the nonborrowing examples, or in both? Also, were they *consistently* superior in either or both types of subtraction example?

To answer these questions the test papers of all pupils in Section NC, numbering 121, were compared with the test papers of an equal sample[1] from Sections R, D, and O. (Up to this time, it will be recalled, all crutch sections had had exactly the same instruction.) The figures for the comparison are presented in Table 8. The papers of Section NC showed 24 incorrect answers for Example 1, as compared with 14 for the sample of crutch children; the

TABLE 8

DISTRIBUTION OF INCORRECT ANSWERS ON TEST I FOR THE 121 PUPILS OF SECTION NC AND FOR AN EQUIVALENT SAMPLE FROM SECTIONS R, D, AND O

Example Number	Section NC	Sections R, D, and O
(1)	(2)	(3)
1	24	14
*3	30	16
4	39	15
5	30	14
*6	20	11
8	39	19
9	39	13
11	38	14
*12	6	4
*14	19	17
15	46	17
16	46	14
*17	31	12
18	43	25
20	45	24
Total Incorrect Answers	495	229
Mean Incorrect Answers per Example	33	15.3
Borrowing Examples		
Total Incorrect	389	169
Mean Incorrect per Example	38.9	16.9
Nonborrowing Examples		
Total Incorrect	106	60
Mean Incorrect per Example	21.2	12

*Examples not requiring borrowing.

[1] That this sample of 121 papers from Sections R, D, and O was a valid one was established by comparing the mean score and the S.D. of the sample with the mean score and the S.D. of the entire sections on the Pretest.

corresponding figures for Example 3, the first of the nonborrowing examples, are 30 and 16 respectively.

It will be observed that the crutch children were consistently superior in every one of the fifteen subtraction examples, regardless of whether borrowing was required. The ratios of incorrect answers for the two groups of children on the ten borrowing examples is 38.9 to 16.9, or 2.3 to 1. The ratios for the five nonborrowing examples is 21.2 to 12, or 1.8 to 1. In other words, while the crutch children did better on the borrowing examples, they also did better on the older type of example in which no borrowing was required. There seems to be some evidence, then, not only that the crutch made for greater accuracy in dealing with the type of example to which it applies (borrowing examples), but also that it *prevented* errors with a type of example to which it does not apply

TABLE 9

DISTRIBUTION OF INCORRECT ANSWERS FOR HIGH AND LOW IQ GROUPS AND FOR HIGH AND LOW ARITHMETICAL ABILITY GROUPS (ACCURACY); TEST I

Example Number	IQ Groups				Arithmetical Ability Groups (Accuracy, Pretest)			
	High (113+)		Low (98—)		High (19+)		Low (14—)	
	NC (N=29)	R-D-O (N=29)	NC (N=26)	R-D-O (N=26)	NC (N=37)	R-D-O (N=37)	NC (N=35)	R-D-O (N=35)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	4	2	6	4	3	2	13	8
*3	7	2	10	5	6	2	13	9
4	9	3	11	8	4	4	22	9
5	8	1	7	1	6	0	16	6
*6	2	1	6	4	4	0	8	4
8	4	1	10	8	4	4	19	9
9	5	1	10	3	3	2	17	10
11	6	3	6	4	6	2	16	11
*12	0	0	4	2	1	0	4	4
*14	6	2	4	8	3	3	10	11
15	6	2	14	6	8	3	21	13
16	9	3	15	3	7	5	22	6
*17	6	0	9	4	4	0	17	11
18	9	7	12	6	8	5	19	8
20	7	7	13	1	1	7	24	12
Totals, Incorrect Answers	88	35	137	67	68	39	241	131
Mean Incorrect Answers Per Example All Types	5.9	2.3	9.1	4.5	4.5	2.6	13.4	8.7
Per Borrowing Examples	6.9	3.0	10.4	4.4	5.0	3.4	18.9	9.2
Per Nonborrowing Examples	3.8	1.0	6.6	4.6	1.8	1.0	10.4	7.8

(nonborrowing examples). Unlike the crutch children, the non-crutch children seem to have become confused in the face of the new process of borrowing and to have tended to make mistakes on examples which they were supposed to have mastered. Other evidence on this point will be presented in Section *3*, below.

c. Relative success of ability groups on Test I

It is not unreasonable to infer that the value of the crutch, as demonstrated in paragraphs *a* and *b* above, may have been limited to pupils of inferior ability. As a matter of fact, opponents of such "learning aids" as the one here under investigation are quite clear in their statements that however useful these devices may be to slow pupils, they are not needed by more capable children. On this account the data assembled in Table 9 are of interest.

In Columns 2 to 5 the relative success of high and of low IQ groups on Test I is open for study. The 29 pupils with the highest IQ's in Section NC are compared with a sample of 29 pupils from Sections R, D, and O who possessed comparable IQ's; and similar groups with low IQ's (below 98) are also compared. As one reads down the pairs of figures for incorrect answers on the fifteen examples of Test I, one is impressed by the fact that in every instance except two (Examples 12 and 20 for the brighter group) the advantage lies with the crutch children. The rows of data at the bottom of the table summarize the figures, but perhaps in form not readily susceptible to comparison; hence the ratios are given below:

Examples (1)	Ratios of Means of Incorrect Answers, for IQ Groups: High NC vs R-D-O (2)	Low NC vs R-D-O (3)
All Examples	2.6 : 1	2.0 : 1
Borrowing Examples	2.3 : 1	2.4 : 1
Nonborrowing Examples	3.8 : 1	1.4 : 1

To consider first the data for the two brighter groups (Column 2): regardless of type of example, the crutch enabled its users to surpass in accuracy the children who had not been taught the device. In terms of mean number of incorrect answers, the ratios are all larger than 2 : 1. Especially interesting is the fact that the greatest difference appears to lie in the nonborrowing examples. While too much weight should not be attached to this comparison because of the small number of examples, it would seem that the crutch offered its greatest help to brighter pupils by holding steady their mastery of subtraction without borrowing. Be that as it may,

these data refute the argument that the crutch could have had little to offer brighter children.

The ratios in Column 3, above, show that the crutch, as expected, did help the slower children. The differences in the ratios as between the crutch and the non-crutch children are, however, not so large as in the case of children of high IQ. Moreover, with the low IQ groups the crutch seems to make its contribution chiefly to success with the new type of example.

Columns 6 to 9 of Table 9 provide a basis for comparing the value of the crutch for children of high initial ability in arithmetic and for other children of low initial ability. Fourteen of the fifteen comparisons in the case of the high groups are in favor of the crutch children (the exception is Example 20), and the same number of comparisons in the case of the low groups favor the crutch children (the exception is Example 12). The crutch then served the interests of both the more capable and the less capable, not the latter alone.

The summary of ratios below is similar to that for the IQ groups just presented. The greater difference in the ratios for the

Examples (1)	Ratios of Means of Incorrect Answers, for Arithmetic Ability Groups: High NC vs R-D-O (2)	Low NC vs R-D-O (3)
All Examples	1.7 : 1	1.5 : 1
Borrowing Examples	1.5 : 1	2.1 : 1
Nonborrowing Examples	1.8 : 1	1.3 : 1

high ability groups is in the case of the nonborrowing examples, and for the low ability groups, in the case of the borrowing examples. These trends agree with those reported for IQ groups, but the differences are not so large.

Conclusion of Section 1

The comparisons reported in the three parts of this section relate to accuracy alone. On this point they show (1) that the crutch made important contributions in the early stages of learning to borrow. The advantage held by the crutch children persisted, in the case of two crutch sections, through Test II, but for all practical purposes was gone when Test III was given. (2) In the early stages of learning the crutch contributed to greater accuracy both (a) by facilitating higher scores on the new kind of subtraction examples and (b) by preventing loss of control over computation

with older kinds of subtraction examples. (3) The usefulness of the crutch was by no means confined to the less able pupils. On the contrary, it seems also to have benefited children with high IQ's and children of superior initial ability in computation. There is some evidence too that it rendered its chief assistance to the brighter and the more capable children by preventing losses on familiar kinds of examples, and to the slower and less capable children by making borrowing easier.

2. EVIDENCE ON RATE OF SUBTRACTION

The question in this section of Chapter III is: Did use of the crutch influence rate of subtracting favorably, unfavorably, or not at all? Again, as in the case of accuracy of work, an answer to the question is sought in a comparison of the scores made by the crutch and the non-crutch children on Tests I, II, and III. Distributions of rate scores are found in Tables 10 and 11.

On the Pretest (Table 5) the four sections were shown to have been equal in their rate of computation; on the average the four sections all completed twenty examples before the time signal was given. The results on Test I (Columns 2-5, Table 10) reveal the initial effects of use of the crutch. Again all sections proved to be equal in rate of work. About two weeks later on Test II, Sections NC and D had pulled away from the other two sections, and by statistically reliable differences (Columns 6-9). Section R was markedly slower on the test than was any other section. Section O also lost ground relatively. The comparative rates of work on the three parts of Test III are given in Table 11.

In order better to show the differences between the crutch and the non-crutch children Figure 3 has been constructed. The rate scores of the various sections on each test have been made comparable by dividing the median rate scores of the four sections by the number of examples in each test. Thus, the medians of 20 on the Pretest have all been divided by 21, as has also been done in the case of Test I and Test II. The median scores on the three parts of Test III have been divided by 13.[2]

Perhaps the most noteworthy fact to be observed in Figure 3 is the exact equivalence of the rate scores on Test I. At first glance this situation would hardly have been expected. After all, the chil-

[2] Comparisons may be made between Test I and II, but not between the parts of Test III and either of the earlier tests. The reason is that while the number of examples and the time limits were the same for Test I and II, they differ as between those tests and Test III.

TABLE 10

DISTRIBUTION OF RATE SCORES ON TESTS I AND II, FOUR EXPERIMENTAL SECTIONS; TWENTY EXAMPLES (BOTH ADDITION AND SUBTRACTION)

Score	TEST I, BY SECTIONS				TEST II, BY SECTIONS			
	NC	R	D	O	NC	R	D	O
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
*21	17	15	13	20	28	13	30	32
20	16	18	19	13	27	11	36	15
19	5	5	6	2	4	2	6	4
18	6	5	8	9	3	6	2	9
17	8	5	11	8	4	4	4	5
16	11	4	8	4	7	8	4	5
15	7	8	7	6	4	7	2	4
14	9	5	4	8	4	10	3	4
13	4	7	7	3	0	6	5	9
12	2	4	3	4	4	7	5	4
11	5	4	5	4	3	1	2	5
10	5	8	4	4	5	1	3	4
9	2	2	1	12	2	2	0	2
8	1	5	4	2	0	5	0	3
7	0	0	0	1	0	3	0	2
6	1	0	1	0	0	3	1	0
5	0	1	0	3	0	2	1	1
Totals	99	96	101	103	95	91	104	108
Medians	17	17	17	17	20	15	20	18
Qs	3.0	3.5	3.0	4.5	2.5	3.5	2.5	3.5
P. E. Mdn.	0.4	0.5	0.4	0.6	0.3	0.4	0.2	0.3

*All pupils who completed the test before the time signal are arbitrarily credited with rate scores of twenty-one examples.

Since all the medians in Cols. 2-5 are the same, the Critical Ratios on Test I are all 0.

The Critical Ratios for Cols. 6-9 are:

NC — R = 10.0 NC — D = 0.0 NC — O = 4.8
R — D = 11.1 R — O = 6.0 D — O = 5.6

dren in the three crutch sections, all taught alike up to this time, had to write out a rather complicated series of operations before subtracting. On the other hand, the non-crutch children could proceed at once to subtract. Yet, the rate scores of both groups of children are the same. The explanation seems to be that *at this stage in learning* it required no longer actually to *make* the changes in the examples than it did to *think through* the process. Viewed this way, the agreements are not so surprising.

After Test I, the children of Sections NC and D made almost identical rate scores. Use of the crutch had now been *denied* the D-children, and they were able to subtract as rapidly as were the children who had never been taught the crutch. This was not true, however, in the case of Sections R and O, whose records will be more fully considered in Section *4* of the chapter.

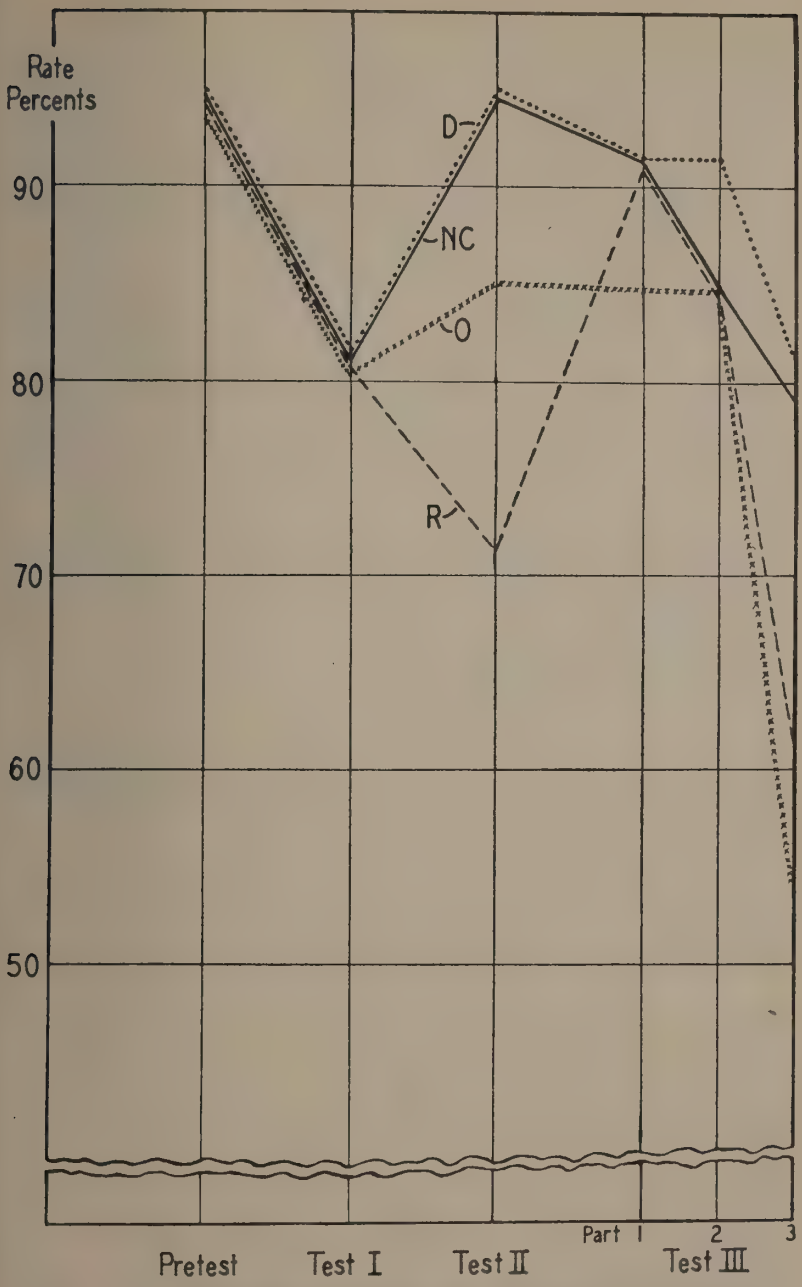


FIG. 3. Rate percentages on various tests, Section NC vs. Sections R, D, and O.

TABLE 11

DISTRIBUTION OF RATE SCORES ON TEST III, FOUR EXPERIMENTAL SECTIONS; TWELVE SUBTRACTION EXAMPLES IN EACH OF THE THREE PARTS OF THE TEST

Score	PART 1, BY SECTIONS				PART 2, BY SECTIONS				PART 3, BY SECTIONS			
	NC	R	D	O	NC	R	D	O	NC	R	D	O
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
*13	33	26	29	27	29	14	34	28	16	1	12	13
12	20	24	28	21	19	25	30	19	14	12	22	9
11	5	4	3	13	6	12	9	10	7	6	12	8
10	7	11	8	8	4	12	7	15	12	10	14	7
9	8	8	8	10	14	10	7	6	7	8	10	13
8	9	5	7	2	13	2	5	4	11	17	13	8
7	8	7	11	8	4	6	7	7	12	18	8	13
6	4	6	4	9	6	7	2	6	11	7	4	8
5	1	1	1	3	0	2	0	3	3	4	3	10
4	2	2	2	2	2	4	0	5	4	11	3	14
Totals	97	94	101	103	97	94	101	103	97	94	101	103
Medians	12	12	12	11	11	11	12	11	10	8	10	7
Qs	2.0	2.0	2.0	2.0	2.5	1.5	1.5	2.0	2.5	1.5	2.0	2.5
P. E. Mdn.	0.3	0.3	0.3	0.2	0.3	0.2	0.2	0.2	0.3	0.2	0.3	0.3

*All pupils who completed a test part before the time signal are arbitrarily credited with a rate score of thirteen examples.

None of the Critical Ratios for Part 1 are reliable.

Two Critical Ratios for Part 2 are reliable:

$R - D = 3.5$; $D - O = 3.5$

Four Critical Ratios for Part 3 are reliable:

$NC - R = 5.6$ $NC - O = 7.2$ $R - D = 5.6$ $D - O = 7.2$

In summary, it may be said that the crutch had no effect at all upon rate of work for a short period after the process of borrowing had been introduced; likewise, that early use of the crutch imposed no later speed handicap upon children (Section D) once they were forbidden to use it; but that later the requirement of regular use (Section R) and the making of its use optional (Section O) both affected rate of work harmfully.

3. EVIDENCE ON UNDERSTANDING OF BORROWING IN SUBTRACTION

The measures thus far presented have been those which describe the *efficiency* of subtraction. That is to say, they reveal how accurately and how rapidly the experimental subjects were able to subtract. But they reveal nothing at all concerning the relative degree to which the crutch and the non-crutch children *understood* what they were doing when they borrowed. It is of course possible to argue that understanding is not an essential aspect of the ability to borrow in subtraction, that all that is necessary is mechanical

proficiency, and that the final test of learning is not whether children see sense in what they are doing, but rather whether they can compute correctly and quickly. Still, even those who deny the importance of an intelligent grasp of the principles of computation will hardly object to a device which yields this product, provided that it imposes no penalty upon efficiency.

Sections *1* and *2* of this chapter have certainly demonstrated that the crutch does not impair accuracy and rate of work. The question for this section, then, is: Does use of the crutch make for a better understanding of the process of borrowing? Evidence of two kinds will be considered: (a) that from test papers, on the comparative susceptibility of crutch and non-crutch children to various errors, and (b) that from teachers, on the usefulness of the crutch for putting meaning into the process of borrowing. The second kind of evidence is drawn from the teachers' logs.

a. Relative susceptibility to errors on Test I

Table 12 contains an analysis of the test papers for Section NC and the sample of Sections R, D, and O used in Table 8. This analysis was intended to reveal types of error made in working the examples of Test I. The first row of the table is read as follows: In Example 1 (the first example requiring borrowing) 15 non-crutch children and 1 crutch child failed to borrow as needed; the same number of combination errors (10) appeared on the papers of both groups; 2 non-crutch children and 3 crutch children made reversal errors (subtracted the minuend figure from the larger subtrahend figure); 2 crutch children made errors peculiar to the use of the device; the totals are 27 errors for non-crutch and 16 for crutch children. The table is divided so as to facilitate comparisons between the groups on borrowing and on nonborrowing examples.

Several facts of importance are to be found in the table. First, a comparison of totals shows that the non-crutch children made 495 errors to the 242 made by the crutch children; the ratio is almost exactly 2:1. On the nonborrowing examples the ratio is 1.2:1 (80:67), and on the borrowing examples, 2.4:1 (415:175). Here again is evidence that Section NC not only found borrowing more difficult than did the crutch children, but also that they seemed to suffer some loss of accuracy in dealing with supposedly familiar kinds of subtraction.

The next comparisons of importance are on the various types

of error listed at the top of the table. The data in Columns 2 and 3 are especially interesting: the non-crutch children were twenty-one times as likely as were the crutch children to fail to borrow when the example called for this process. Unmistakably, the crutch was

TABLE 12

TYPES OF ERROR IN SUBTRACTION EXAMPLES, TEST I; SECTION NC COMPARED WITH A COMPOSITE OF SECTIONS R, D, AND O

Example Number	Failure to Borrow		Unnecessary Borrowing		Added One or More Digits		Combination Errors		Reversals		Crutch* Errors	Totals	
	NC	R-D-O	NC	R-D-O	NC	R-D-O	NC	R-D-O	NC	R-D-O	R-D-O	NC	R-D-O
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
Borrowing													
1	15	1	..	..	..	..	10	10	2	3	2	27	16
4	24	..	..	..	2	..	13	9	7	5	3	46	17
5	17	..	2	..	..	1	6	3	5	4	3	30	11
8	26	2	..	..	1	1	8	6	10	9	2	45	20
9	18	..	..	..	1	..	11	5	8	5	3	38	13
11	25	..	1	..	..	..	7	10	7	4	2	40	16
15	29	..	..	..	1	..	7	6	9	10	2	46	18
16	30	1	1	..	..	..	9	3	9	7	2	49	13
18	24	2	1	..	3	..	9	10	9	12	3	46	27
20	26	5	1	..	1	..	9	10	11	7	2	48	24
Totals	234	11	6	..	9	2	89	72	77	66	24	415	175
Nonborrowing													
3	..	..	5	7	3	2	25	9	..	..	1	33	19
6	..	..	4	7	2	1	16	3	..	..	..	22	11
12	..	..	3	3	1	..	1	1	..	..	1	5	5
14	..	..	5	8	..	..	15	8	..	..	1	20	17
17	..	..	1	4	..	..	22	5	10	4	2	..	15
Totals	..	..	18	29	6	3	79	26	10	4	5	80	67
Grand Totals	234	11	24	29	15	5	168	98	87	70	29	495	242

*The "Crutch Errors" were limited of course to Sections R, D, and O; they represent errors which resulted from imperfect mastery of the crutch itself. They are: (1) Subtracted the crutch "1" from the subtrahend digit (8 cases); (2) Failed to subtract from one figure in the minuend (2 cases); (3) Failed to subtract after altering minuend figure (1 case); (4) Separated tens figures into two columns because of unnecessary borrowing (6 cases), and (5) Made a subtraction error in altering minuend figure (12 cases).

of value to its users: its use revealed at once the need for borrowing, a need which was by no means clear to the non-crutch children. On the other hand, the use of the crutch tended, according to Columns 4 and 5, to make its users prone to borrow when this was not necessary. The difference between the sections is, however, immaterial since the number of such errors is small. Likewise, the data in Columns 6 and 7 may be passed over.

Columns 8 and 9: The non-crutch children made 1.7 combination

errors for each such error made by the crutch children. The ratios in the borrowing examples are 1.2:1, and in the nonborrowing, 3:1. Here then is another striking difference between the two groups of children. Once more the behavior of the non-crutch children reveals disturbance of a serious character in the face of a new and none-too-well understood procedure. The crutch children made an average total of 7.2 combination errors per example in the borrowing examples as compared with 5.2 errors in the nonborrowing examples. This variation is in the expected direction. On the other hand, the non-crutch children made 15.8 errors on the latter kind of example, the familiar kind, as compared with 8.9 such errors in the new type; and this condition is hardly the expected one. Apparently combinations previously learned simply would not "work" correctly in the general confusion aroused by an unintelligible task.

The data in Columns 10-11 on Reversals manifest no important differences between the two groups of children. The figures in Column 12 show that the crutch children made a total of 29 errors peculiar to the use of the crutch, these errors being distributed among five types. While there is no point in ignoring the existence of these errors—they must be charged up against the use of the crutch—their number is small and argues for dismissal with but a word. The most frequent of the errors occurred twelve times; it consisted in a subtraction error in altering the minuend. The teacher who teaches the crutch may well be warned against the probable but infrequent appearance of this error. In a word, this error (and the others listed below the table) are largely of significance to the teacher who decides to teach the crutch as an initial aid in borrowing.

To conclude this discussion of Table 12: here are found the reasons for the consistent superiority of the crutch children on Test I. In the first place, use of the crutch seems to have insured borrowing when borrowing was necessary (and thus explains the advantage of the crutch children on the borrowing examples of Test I). In the second place, it made more sensible the general procedure of borrowing, and so prevented confusion and loss of skills already developed (and so explains the superiority of the crutch children on the nonborrowing examples).

b. Teachers' opinions of the crutch as an aid to understanding

The teachers of Sections R, D, and O were never directly questioned concerning their views of the worth of the crutch. The fol-

lowing voluntary statements, taken for the most part from their logs, are therefore the more significant.

Teachers of R-sections:

1. "The children liked the crutch. One child said, 'I like this kind.' Another said, 'Isn't it easy?' Still another thought it made subtraction easier." This teacher reported also that once children had learned the crutch, there was a tendency for them to become more interested in checking answers. The reason was not given.

One interesting incident occurred in this class. A newcomer in the group worked an example without the crutch. The other children said that the solution was "wrong." The teacher then showed the solution to be correct. "Then the children, some of them, began to try their hand in working without the crutch, but they *forgot to lessen the 'top number' by one.*" In other words, they needed the services of the crutch.

2. Use of the crutch "helped the slower pupils amazingly. Only one or two failed to get the first example correct."

3. "I had no trouble getting the children to use the crutch. It seemed to them to be a perfectly natural thing to do."

4. "I encountered little trouble in explaining borrowing with the help of the crutch. . . . Its use made for greater accuracy of work than had ever before been shown by any of my classes."

Teachers of D-sections:

1. "The pupils followed the crutch method very well except for four slow, failing children."

2. "The crutch helped the children to understand borrowing, especially the slower children."

3. "I have always used this crutch in teaching borrowing, and have found it to be helpful. . . . The pupils learned it easily and seemed to understand very well what they were doing." Later, at the time of Test IV: "The value of the crutch for my teaching is that I have found that subtraction mechanics are more readily understood with its use in introducing borrowing."

4. "The children learned the crutch easily . . . but some got so absorbed in marking out numbers that they didn't look to see if this was necessary."

Teachers of O-sections:

1. "The children caught on to borrowing very easily by using the crutch."

2. "The children learned to borrow easily, and made few mistakes from the start." Later, at the time of Test IV: "I do not think any child has been hurt by it. I am inclined to think the weak children were benefited."

3. No log report, but this statement at the time of Test IV: "I am positive that teaching children to subtract with the use of the crutch makes borrowing much easier for them."

4. "I expect always hereafter to teach borrowing with the crutch."

There seems to be general agreement among the teachers that the crutch was of service in explaining borrowing. On this, the teachers' opinions confirm the data presented earlier in the chapter, except at one point. The teachers rather imply that the crutch was useful chiefly, if not wholly, for the slower pupils, whereas it has been shown to have been useful to all levels of ability. It may well be that the teachers merely were more sensitive to the needs of the less capable children, or that the latter pupils more obviously revealed the help they found in it.

The weight which will be allowed this evidence from teachers will depend upon the reader's evaluation of teachers' opinions. In an investigation of this kind unsolicited support for an experimental device seems, to the writers at least, to be of considerable importance.

4. VALUE OF THE CRUTCH UNDER VARYING CONDITIONS OF INSTRUCTION

Thus far in the chapter the discussion has been held as closely as possible to comparisons between the crutch and the non-crutch children. In the following paragraphs attention will be given to the three crutch sections which were taught exactly alike until immediately after Test I. From that time on, Section R continued as before to use the crutch under specific directions from the teachers to do so in all assigned subtraction examples which required borrowing. Section D, however, was denied use of the crutch. While no penalties were to be applied (in order to avoid unwholesome emotional conflicts), these children clearly understood that they were to abandon the crutch. In the meantime, the children of Section O had been shown how to borrow without the crutch and they were allowed to adopt the shorter form if they wished to, *but only if they wished to.*

Obviously, these differences in instruction for Sections R, D, and O were introduced for a purpose. One reason was to study the effect of teachers' attitudes toward the crutch on the persistence or disappearance of the device. This problem will be treated in the next chapter. Another reason was to ascertain the effect of differing teachers' attitudes toward the crutch on efficiency (accuracy and rate) of subtraction. This latter problem is the subject to be considered here. Stated as a question, the problem is: Supposing that the crutch is taught in introducing the process of borrowing, should its later use be insisted upon, or should it be prohibited, or should its use be left to the desires and preferences of the children them-

selves? In answering these questions the reader will find especially helpful Figure 2 on page 30 and Figure 3 on page 37. Of necessity, the discussion of these figures and of the data upon which they are based will duplicate much that has been said in the foregoing sections of this chapter. The casual reader, therefore, may prefer to skip the next twelve paragraphs and to read only the "conclusion" beginning on page 46.

Variation in accuracy

Generally speaking, Section R made the best accuracy record of the three crutch sections. On Test II the children of this section increased their accuracy by a substantial margin. This gain is of course not surprising: after all, if the crutch helped at all so far as accuracy is concerned, continued use of it for a time should have had just this effect. On Part 1 of Test III the R-section children maintained a high degree of accuracy, though there was some loss from Test II. A larger loss occurred on Part 2, and a still larger loss on Part 3. The loss on Part 2, when they were told that they might not use the crutch, may mean that most of the children were still dependent upon the device and could not get along without it. Or, it may mean that even for the children who had discarded the crutch the unusual prohibition on the part of the teacher disconcerted them and lowered the quality of their work. The solution of the problem thus lies in their reliance upon, or their freedom from, the crutch at this stage in their learning. Data on this point will be presented in the next chapter.

Next to Section R, Section D made the best accuracy record of the crutch sections. On Test I these children equalled the median accuracy score of Section R. When then they were denied further use of the crutch, they maintained the quality of their work (Test II), though they failed to show any gain in accuracy, as was the case with Section R. The fact that on Part 1 of Test III they were able to raise their accuracy score may be interpreted as meaning that they had been forbidden use of the crutch a little too soon, so that only by rather extended practice were they able to reach what may be considered a satisfactory level of accuracy.

The reactions of the D-section children to Part 2 of Test III are interesting. These children, like those in Sections R and O, were told that they could not use the crutch for the twelve examples in this part. In spite of the fact that by this time they had almost unanimously gotten away from the crutch, and in spite of the fact

that they were subtracting in examples of a familiar type, these children showed a loss of two points in accuracy percentage.

The records of the O-sections are ambiguous. The children of these sections were apparently not the equal of those in the R- and D-sections. On the Pretest Section O had the poorest accuracy mean of any of the four experimental sections, it being 1.3 less than the mean of Section R and .8 less than that of Section D. The first of these differences was nearly reliable (Critical Ratio = 2.9). On Test I, while the three crutch sections were still being taught exactly alike, its mean score was 12.0 as compared with 12.9 for both Section R and Section D. The apparent weakness of Section O makes it impossible to interpret relative changes in accuracy as purely the result of differences in instruction with respect to the crutch.

The children of the O-sections were allowed to decide, after Test I, whether they would retain or reject the crutch. As will be shown later, they tended steadily to abandon the device. In doing so, however, they failed as a section to show any increase in accuracy; instead, their accuracy means were one percentage point less on Test II than on Test I, and one percentage point less on Part 1 of Test III than on Test II. On Parts 2 and 3 they showed the same lowering of quality of work which characterized all other sections, but the loss on Part 3 was of great size.

It is rather dangerous to hazard generalizations regarding the comparative accuracy of the three crutch sections. The facts for Section R seem to suggest that long-continued use of the crutch has few if any harmful consequences on accuracy. The facts for Section D may mean that it is possible to deny use of the crutch too soon, and the facts for Section O may mean that indecisiveness with regard to use of the crutch (that is, absence of specific directions from the teacher) may impair efficiency.

Variation in rate

For the study of differences in rate of work among the crutch sections Figure 3, page 37, summarizes the facts.

The three crutch sections, being taught alike at first, showed exactly the same rate of work on Test I, and the same rate of work as was exhibited by the non-crutch children. This unexpected agreement of crutch and non-crutch children has already been commented upon (see pp. 35 and 36).

On Test II, Section D, denied the crutch, showed a great gain

in rate of work, as might have been anticipated. Section R, still using the crutch, would hardly have been expected to show much of a gain in rate, but they could not at all have been expected to show a great loss in rate, as was the actual fact. Section O, left to decide for themselves whether they would use the crutch, showed a slight increase in rate, which can be thought of as due to the rather uncertain abandonment of the device by some of the children.

On Part 1 of Test III, the children of Section R showed a remarkable recovery, equalling in rate the record of Section D. On the supposition that the R-children were still using the crutch, as they were directed to do, this gain in rate is difficult to account for. If, however, the children of this section were surrendering the device (and this was the case), their rate gain is more understandable. Unlike Sections R and D, Section O proceeded to work at the same pace as before.

Denial of the use of the crutch on Part 2 had no effect on the rate of Section O, or of Section D (the latter having long been accustomed to work without it), though as shown in Figure 1 both sections suffered in accuracy of work. In Section R apparently enough of the children were still dependent upon the crutch to be affected harmfully by its prohibition.

All three crutch sections naturally were slowed up by the new examples of Part 3, the effect being greatest on Sections O, R, and D in the order given.

Conclusion

If the data on relative rate of work are placed alongside of those on relative accuracy of work, the effects of varying attitudes toward continued use of the crutch may be noted. (1) If the crutch is taught and then is required for a comparatively long period of time (Section R), a high degree of accuracy may be expected, though for obvious reasons not as many examples will be done in a given period of time. (2) If the crutch is taught and is then denied (Section D), there will not at once be improvement in accuracy, but there will be an understandable gain in rate of work. (3) If the crutch is taught and then its use is made optional (Section O), there seems to be little effect either on accuracy or on rate of work. The third conclusion is questionable, however, by reason of uncertainty as to the actual equality of Section O with Sections R and D.

CONCLUSION OF CHAPTER III

In this chapter two types of comparison have been reported, namely, comparisons between crutch and non-crutch children, and comparisons among the three different groups of crutch children. Most of these comparisons have been made in terms of efficiency of work.

1. With regard to accuracy, the crutch has been shown to have made important contributions in the early stages of learning to borrow. That is, at first the crutch children as a group were able to secure more correct answers in subtraction than were the non-crutch children. In the case of two of the crutch sections this accuracy advantage was held, though with lessening degree, to the end of the experiment.[3] Moreover, the contribution of the crutch was (a) consistent and (b) general. It was consistent in that it enabled its users to be more accurate both with borrowing and with nonborrowing examples, probably by enabling them better to discriminate between the two types of example. It was general in that its beneficial effects were experienced both by the bright and by the slow, and both by those superior and by those those inferior in arithmetic ability.

2. With regard to rate of work, the evidence is that at first the crutch imposes no handicap. On the contrary, apparently at this time the thought processes involved in borrowing mentally, as was done by Section NC, are as complicated, cumbersome, and time-consuming as are the activities of making actual changes in the minuends. As non-crutch children became more familiar with the mechanics of borrowing, they overtook and surpassed in rate of work the children who continued to use the crutch, but not those who for one reason or another had discarded it. That is to say, initial use of the crutch is not uneconomical; nor does it have detrimental effects on rate which persist after its use has been discontinued.

3. Comparisons between crutch and non-crutch children were also made on the basis of their relative understanding of the process of borrowing. On this point some of the findings have already been suggested in paragraph 1 above: the children who used the device not only were able to make higher scores on borrowing examples,

[3] The record of the third crutch section, Section O, is quite different from the records of the other two, a condition which may in part be due to corresponding differences in teachers' attitudes toward continued use of the crutch, but which is probably more largely due to general inferiority in arithmetic.

but they were protected from loss of efficiency with familiar non-borrowing examples. Confirmation was given this conclusion by the results of comparing the errors made by children in the NC-section with those made by a selected sample of R-, D-, and O-section children. It was found that the non-crutch children failed to borrow when they should have twenty-one times as often as did the crutch children, and that in their general confusion attendant upon learning a but slightly understood process, the former lost control even over the basic subtraction combinations which they were supposed to have mastered.

4. The relative success of the three crutch sections in learning to borrow has been so recently considered and summarized as to make unnecessary a repetition of the findings at this point.

In general, the showing made by the crutch was surprisingly good. As judged in the usual terms of efficiency, or in the less usual terms of understanding, the crutch seems unmistakably to have benefited its users. Even the common objection that such a device seriously and permanently reduces rate of work appears not to be grounded in fact in this instance. If a decision had to be reached as to whether this particular crutch should be taught, the answer would have to be in the affirmative. But there is one important aspect of the problem which has not yet been considered, namely, whether the crutch persisted as a habit in the subtraction behavior of its users. Evidence on this question will be adduced in Chapter IV. And in Chapter V the significance of the findings reported in this chapter will be considered—its significance both for the teaching of arithmetic and for learning theory in general.

CHAPTER IV

PERSISTENCE OF THE CRUTCH

Common arguments against crutches can be grouped under two heads: (1) they do not facilitate learning; indeed, they may be as difficult to learn as is the process they are supposed to simplify; and (2) they remain as fixed habits to interfere permanently with efficient performance. In so far as these objections apply to the subtraction crutch here under investigation, the first group of objections have been met. It remains to determine whether the crutch tended to persist or tended to disappear.

The answer to the question (like the answers to the questions raised in Chapter III) is significant not merely for arithmetic instruction, but for valid learning theory. If this crutch should be found to have long outlasted its usefulness, there would be grounds, not only to oppose teaching it in arithmetic, but also to insist even more strongly than has been done upon the acceptance and application of a certain view of the learning process. According to this view, we should always, and only, learn precisely what we intend later to do. The character of the ultimate reaction therefore should set the reaction pattern for the beginner; the beginner should adopt this pattern at the outset and practice it unswervingly until he has mastered it. Naturally, he will not be exposed to other less mature, less adult-like patterns, for then, just as naturally, he will practice these less desirable responses and may never attain the final pattern, or attain it only at tremendous expense.

According to this conception of learning, work procedures of the type represented by the subtraction crutch are viewed with alarm, and quite reasonably so. They certainly do not resemble the final pattern, anything but that; and therefore practice on them can only be injurious. Habits will be formed which must later be broken, and this breaking of habits, if accomplished at all, is done at a cost for which there is no compensation. The possibility that children may grow away from these immature habits is not to be considered seriously. Indeed, the question may be asked: *Why* should children do anything of the kind? What is there about the crutch, or about the child, or about his development which should make the learner abandon a device which he has mastered? Does not the Law of Exercise (under any of its names) unequivocally predict that what

is practiced is learned? How then can practice on a form of behavior contribute to its eventual disappearance?

And so, it is worth while to see what happened to this subtraction crutch. Did the children tend to give it up? If they did, or some of them did, under what conditions, and what kind of children discarded it? Did it tend to appear regularly or irregularly in late work? If so, under what conditions? These and other related questions are the topics in this chapter, which is divided into four sections:

1. Retention of the crutch (p. 50);
2. Characteristics of short-time and long-time users of the crutch (p. 55);
3. Difficulty of abandoning the crutch (p. 59); and
4. Conditions of late occurrence of the crutch (p. 64)

1. RETENTION OF THE CRUTCH

It can be stated at once and definitely that the crutch did not persist to the degree which opponents of such devices have led us to expect. On the contrary, the tendency to use it decreased steadily from the time when it was taught. Two tabulations reveal this fact clearly.

Per cents of children not using crutch

Table 13 contains one kind of tabulation. To secure the figures of the table, the practice pages and the test papers of each child in the crutch sections were analyzed to see whether he had or had not used the crutch. He was counted as having used it if the device appeared even in a single example and even if it appeared in some shortened form (such as alteration of the tens' figure without insertion of the small "1" before the ones' figure). It should be noted that the per cents are for children who *did not use* the crutch.

Having been told, after Test I, that they could no longer use the crutch, the children of Section D naturally tended to discard it, but the degree of completeness with which they did this is little short of amazing. The records of the other two sections are no less interesting. By the time they took Test III, 17 out of 20 children in Section O were no longer using the crutch; 86.4 per cent had given it up voluntarily. At the same time even the R-children were tending to get along without it, for 7 out of 20 no longer used it.

TABLE 13

PER CENTS OF CHILDREN WHO DID NOT USE THE CRUTCH ON PRACTICE PAGES AND ON TEST I DURING THE EXPERIMENT PROPER

Measuring Instrument	PER CENTS NOT USING THE CRUTCH, BY SECTIONS		
	R	D	O
(1)	(2)	(3)	(4)
Practice, Page I	0.9	0.0	0.9
Test I	6.2	0.9	6.3
Practice, Page II	1.8	100.0	70.5
Test II	12.6	99.1	81.0
Test III, Part 1	37.7	100.0	86.4

Total uses of the crutch

The figures in Table 14, even better than those in Table 13, testify to the extent to which the crutch was abandoned. In Table 14 the unit is the individual example instead of the individual pupil. The per cents reveal the degree to which the crutch was employed at six different times between the early part of December and the middle of May.

Practice Page I consisted of twenty textbook examples which were solved as seatwork immediately after the crutch was taught. All the examples called for borrowing. When the examples were assigned, the children were not instructed whether they should or should not use the crutch. Suggestions on this point were carefully avoided. Nevertheless, it was supposed that they would use it since they had been required until then to do so in all their work. And this is what they did. The children in Section R used the crutch, in whole or in part, in 96.6 per cent of the examples;[1] the children in Section D, in 99.5 per cent; and those in Section O, 93.8 per cent.

Test I was given at the end of the period in which the crutch was regularly required in daily work. As in the case of Practice Page I (and as was also true of later tests and of Practice Page II) nothing was said about the crutch. Between 87 per cent and

[1] To obtain the per cents in the table the number of pupils in each section was multiplied by the number of borrowing examples. The product so obtained then became the denominator of a fraction, the numerator of which was the actual number of uses of the crutch, either the whole crutch or some part of it. This method of calculation is not completely satisfactory, for it neglects the few instances in which the crutch was used unnecessarily; that is, when borrowing was not required. The method, however, has the advantage of including all pupils on each measuring instrument, even though the number varied slightly from measure to measure.

TABLE 14

PER CENTS OF POSSIBLE USES IN WHICH THE CRUTCH WAS ACTUALLY USED

Measuring Instrument	PER CENTS OF ACTUAL USE OF THE CRUTCH, BY SECTIONS					
	R		D		O	
	Whole	Partial*	Whole	Partial*	Whole	Partial*
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Practice, Page I	96.6	0.0	97.7	1.8	93.7	0.1
Test I	86.6	0.4	92.0	1.3	84.5	2.8
Practice, Page II	97.0	0.2	0.0	0.0	22.7	1.1
Test II	87.0	1.6	0.1	0.0	13.6	4.9
Test III, Part 1†	61.6	0.3	0.0	1.1	12.8	2.6
Test IV, 18 Examples‡	12.2	0.7	0.5	0.8	4.8	1.2

*By "Partial" is meant that some part, and not all, of the crutch was used. For instance, the tens' figure might be altered without inserting the small "1" before the ones' figure.

†Only Part 1 of Test III was given under conditions comparable with those for the other measuring instruments. On Part 2 all children were denied use of the crutch; in Part 3 they worked unfamiliar examples.

‡The last column of six hard examples is omitted. Furthermore, data for three crutch sections are omitted: one R-class because the teacher used the crutch to explain multiple borrowing; one D-class because the teacher failed to oppose use of the crutch when it appeared in connection with division; and one O-class because the teacher consistently used the crutch on the blackboard for multiple-borrowing examples.

93.3 per cent of all the examples were worked with the crutch. The children in all crutch sections used the device at nearly every opportunity.

After Test I only the pupils in R-classes were directed to continue use of the crutch. At once pupils in other sections began to desert it: those in Section D because they were denied it, and those in O-classes because they chose to do so. For Section D the per cent fell from 93.3 (whole and partial uses combined) on Test I to 1.1 on Test III; for Section O, from 87.3 to 15.4. Even the R-children tended away from its use; their corresponding per cents were 87.0 and 61.9.

The situation at the time of Test IV (four months after Test III) is to be seen in the figures in the last row of the table. It is hardly too much to say that, for all practical purposes, the crutch was gone in Section D (at least in examples in which borrowing was restricted to tens); that it was about gone in Section O, and that its disappearance was imminent in the case of Section R.

Graphic representation. For readier comparison the data in Tables 13 and 14 are presented in Figures 4 and 5. Figure 4 is based on Table 13, but the percentages have been changed to represent those who used the crutch instead of those who did not. In

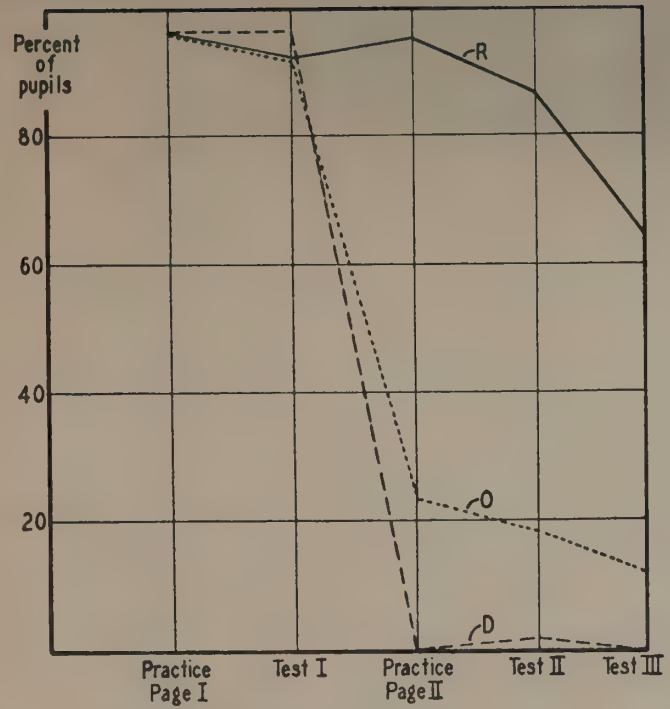


FIG. 4. Per cents of pupils using crutch at successive times of measurement.

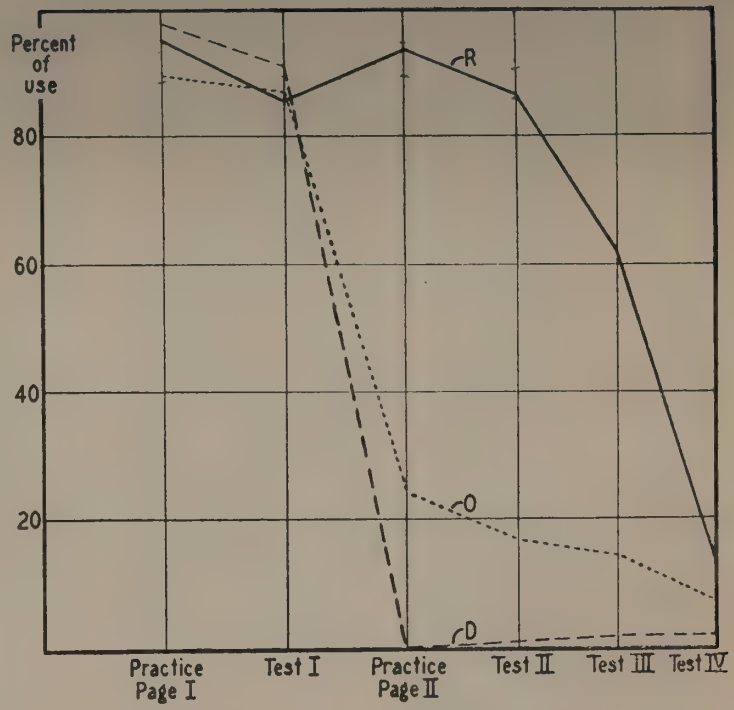


FIG. 5. Actual number of crutch uses divided by possible number of uses, at successive times of measurement.

Figure 5, based on Table 14, the percentages of use of the whole and of part of the crutch have been combined.

Conclusion

The data in Tables 13 and 14 and in Figures 4 and 5 indicate that the assumed dangers of teaching the crutch have been greatly overrated: children who are taught the crutch clearly reveal the tendency to abandon it of their own volition. This conclusion is based upon the records of children in both Section R and Section O. These same records, however, show that not all children will voluntarily drop the crutch; rather, a large number of children who are allowed to adopt a shorter form (Section O) and a still larger number of children who are told to use the crutch will continue to depend on it. The safest policy, so far as discontinuance of the crutch is concerned, seems to be that which was followed in Section D: after the crutch has served its purposes,[2] to prohibit its further use. Under these conditions practically all children may be expected to discard the device.

Relations to efficiency measures (Chapter III)

Some of the questions left unanswered in Chapter III can now be dealt with. One of these questions has to do with the unexpected behavior of the R-children on Test III. From a very slow rate of work on Test II, these children changed to a high rate which was equal to that of other sections. The explanation lies in the fact that 87.4 per cent of these children used the crutch on 88.6 per cent of the possible occasions in Test II. These figures dropped in Test III to 62.3 per cent and 61.9 per cent respectively. Their rate increased, then, because they were not all using the crutch as they were supposed to be doing.

Nevertheless, enough of them (37.7 per cent) still relied upon the crutch at times, so that when in Part 2 of Test III they were told they might not use it, they were somewhat handicapped. This condition is revealed in the decrease in accuracy from Part 1. The knowledge that the crutch was still being used at this time by more than a third of the R-section serves, then, to explain another problem which in Chapter III could be only tentatively solved.

[2] As pointed out earlier (p. 44), this time should actually come later than it did come in Section D in this experiment, perhaps after Test III.

2. CHARACTERISTICS OF SHORT-TIME AND LONG-TIME USERS OF THE CRUTCH

The facts presented in Section *1* show that while there was a rather general shift away from the crutch, the tendency was neither unanimous nor uniform in rate. That is to say, some children apparently remained dependent upon the crutch up to the end of the experiment, and others who got away from it did so at varying rates. It is of interest then to inquire into differences between the children which may possibly explain these variations. Two kinds of data have been tabulated.

Evidence from test papers, through Test II

The first body of data relates to use of the crutch through Test II, in other words, to what may be called the early part of the experiment. First, all pupils in Sections R and O who on Practice Page II and on Test II consistently (always) omitted the crutch were classified together. So also were all pupils in these sections who omitted the crutch generally but irregularly. These two groups could then be compared with their sections as wholes in terms of CA, MA, IQ, and initial arithmetical ability (accuracy and rate on the Pretest).

Table 15 contains the data for Section R. Since but four children omitted the crutch consistently and but six irregularly, they are lumped together here for comparison with the section means and medians. The last two rows of measures indicate that the children who at this time were abandoning the crutch most noticeably were somewhat older than the average child in the section, but less mature mentally and of lower IQ, though on the Pretest they were about the equal of the section as a whole. The tendency to desert the crutch in the face of the teacher's directions to use it does not seem to be accompanied by outstanding superiority in any of the categories on which personal data are presented in the table. On the contrary, a study of the comparative scores of this group and of the section as a whole on Test II shows that these children may well have abandoned the device too soon: their mean accuracy score on this test was 10.0 as compared with the section mean of 13.6.[3]

Table 16 contains the corresponding data for Section O. Here the children who voluntarily gave up the crutch were, as compared with the section as a whole, slightly older, of about the same men-

[3] Failure to use the crutch of course gave them some advantage in speed, for their rate median was 16.5 as compared with the section median of 15.0.

TABLE 15

SECTION R: COMPARISON OF CHILDREN WHO OMITTED THE CRUTCH CONSISTENTLY OR IRREGULARLY IN EARLY PART OF EXPERIMENT, WITH SECTION AS A WHOLE

Pupils	CA	MA	IQ	Pretest Scores	
				Accuracy	Rate
(1)	(2)	(3)	(4)	(5)	(6)
A. Consistently					
NB	...	9–6	...	20	21
RH	10–2	9–5	93	18	21
PH	9–5	9–7	102	20	21
DS	8–4	9–1	109	18	20
B. Irregularly					
AA	8–5	7–1	84	15	20
PB	....	8–9	....	17	21
FJ	9–10	8–11	91	19	11
DV	9–7	8–10	92	18	20
HH	9–1	8–11	98	10	21
EN	8–4	8–10	106	17	21
Means or Medians					
A and B Combined	9–1.8	8–11	96.9	17.2	21
Section	9–0	9–1.2	102.2	17.3	20

TABLE 16

SECTION O: COMPARISON OF CHILDREN WHO OMITTED THE CRUTCH CONSISTENTLY OR IRREGULARLY IN EARLY PART OF THE EXPERIMENT, WITH SECTION AS A WHOLE

Pupils	CA	MA	IQ	Pretest Scores	
				Accuracy	Rate
(1)	(2)	(3)	(4)	(5)	(6)
A. Consistently					
JH	9–5	9–0	95	19	21
FJ	8–7	8–9	102	14	20
B. Irregularly					
PH	8–2	8–10	107	15	21
CC	8–5	9–10	102	17	21
JB	9–3	10–0	108	20	13
GH	8–8	7–11	91	9	21
AC	10–11	9–10	90	11	20
JH	8–3	9–2	109	9	15
OK	8–11	8–5	95	7	12
EM	9–2	11–1	118	17	20
ZP	8–0	10–0	120	6	12
JS	9–1	8–2	91	9	11
HW	10–11	8–11	80	12	15
Means or Medians					
A and B Combined	9–1	9–2.6	100.5	12.7	20
Section	9–0	9–2.3	103	16.0	20

tal maturity, slightly less bright, and considerably less accurate on the Pretest. On Test II their mean accuracy score was 7.3 and their median rate score was 13, as compared with 11.8 and 18 for the section as a whole. As in the case of Section R, it may be argued that the children in Section O who were most anxious to give up the crutch might well have used it a bit longer.

The data for Section D were tabulated in a slightly different manner. Since D-children were refused the crutch after Test I, a search was made for pupils who nevertheless *used* (not omitted) the device through Test I. No child was found to have used the crutch consistently, and only two who used it commonly but irregularly. Obviously, data for so few children make generalizations impossible, but the data are reported for whatever they may be worth:

	CA	MA	IQ	Pretest Accuracy	Rate
MH	9–7	8–11	93	19	21
JW	9–3	8–2	88	18	20
Section Mean or Median	9–1	9–3.1	102.3	16.8	20

To conclude: *1.* In the early stages of instruction following immediately upon the learning of the crutch the children who most completely abandoned the device (or retained it, as in Section D) were not markedly older, or more mature mentally, or brighter, or more proficient in general arithmetical ability. In other words, eagerness to get rid of the crutch (or to retain it needlessly long) is in itself no evidence either of superior arithmetical skill or of superior general ability. Instead, the reason for the behavior of these atypical children seems to lie in characteristics not here measured, possibly characteristics of temperament. Thus, in the case of children who got rid of the crutch rather early the characteristic might be independence, or impatience with seemingly useless routine steps. The corresponding characteristic of the complacent users of the crutch might be docility, the disposition to do as told whether or not a reason is given. Obviously, no pronounced relation could be expected between such traits and arithmetic ability. *2.* As a rule, children who immediately abandoned the crutch were not successful in borrowing on Test II, a fact which suggests that they actually needed the aid furnished by the device longer than they were willing to use it.

Through Test III

Correlative with the data just presented for the experimental period through Test II are other data based upon the use of the crutch on Test III, and so, some two weeks after the conclusion of the experiment proper. Two tabulations are to be presented.

Table 17 contains a distribution of the pupils in Sections R and O according to (a) IQ and (b) frequency of use of the crutch on the twelve examples of Part 1 of Test III, in nine of which borrowing from tens was called for. One interesting fact is that the pupils in both sections tended either to use the crutch not at all, or to use it in three or more examples.

The last three rows of data in Table 17 reveal no differences in IQ either in Section R or in Section O between children who used the crutch in 0, in 3-8, or in all 9 borrowing examples. Clearly, whatever factor was responsible for continued use or abandonment of the crutch, that factor was not brightness.

Table 18 is similar to Table 17, except that the classification is in terms of accuracy scores on the Pretest instead of IQ. More-

TABLE 17

FREQUENCY OF USE OF THE CRUTCH ON TEST III, PART 1, ACCORDING TO IQ: SECTIONS R AND O

IQ	NUMBER OF EXAMPLES IN WHICH CRUTCH WAS USED, PART 1, TEST III							
	Section R				Section O			
	0	1-2	3-8	9	0	1-2	3-8	9
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
122	1	..	..	..	1	..	..	..
118	3	..	..	..	5	..	..	1
114	6	..	2	3	13	..	1	2
110	6	..	1	5	14	..	..	2
106	7	..	2	14	15	..	2	1
102	4	..	3	6	13	..	1	2
98	6	..	1	6	1	..	..	1
94	3	..	..	7	9	..	1	..
90	2	..	1	4	7	..	..	..
86	1	..	..	1	11	..	..	..
82	2	..	..	1	..	..	..	1
78	..	..	..	2	1	..	..	..
74	..	..	..	..	1	..	..	..
Total	41	0	10	49	91	0	5	10
Mean IQ	104.0	..	104.0	104.0	102.4	..	104.4	105.6
$\sigma_{dist.}$	9.8	..	10.8	8.8	10.7	..	6.5	9.9
σ_M	1.5	..	3.5	1.3	1.1	..	3.0	3.2

over, the conclusions to be drawn from this tabulation are similar to those for the IQ tabulation: differences in the degree to which the crutch was used in Test III are not to be accounted for in terms of differences in initial arithmetical ability (accuracy scores on the Pretest). The two tables, then, confirm the suggestion that the tendency to rely perhaps overlong on the crutch is the result of temperamental rather than of arithmetical or intelligence factors.

TABLE 18

FREQUENCY OF USE OF THE CRUTCH ON TEST III, PART 1, ACCORDING TO INITIAL ACCURACY: SECTIONS R AND O

Pretest Accuracy Score	Number of Examples in Which Crutch Was Used, Part 1, Test III							
	Section R				Section O			
	0	1-2	3-8	9	0	1-2	3-8	9
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
20	8	..	..	15	12	..	..	..
19	10	..	2	9	20	..	1	2
18	5	..	2	2	9	..	..	2
17	6	..	2	8	9	..	..	2
16	2	..	..	5	7	..	..	2
15	3	..	..	..	9	..	3	..
14	..	..	1	6	5	..	..	..
13	..	..	..	1	4	..	..	..
12	2	..	..	1	4	..	..	..
11	..	..	..	..	4	..	..	..
10	..	..	..	1	1	..	..	1
9	1	..	..	1	3	..	..	..
8	..	..	..	..	..	..	..	..
7	..	..	..	..	..	..	1	..
6	..	..	..	..	1	..	..	..
5	..	..	1	..	1	..	..	..
Totals	37	..	8	49	89	..	5	9
Mean Score	17.6	..	15.9	17.4	16.2	..	14.2	16.7
$\sigma_{dist.}$	2.5	..	4.4	2.7	3.4	..	3.8	2.5
σ_M	0.4	..	1.6	0.4	0.4	..	1.7	0.8

3. DIFFICULTY OF ABANDONING THE CRUTCH

Just how hard did the children of Sections R, D, and O find it to give up use of the crutch? Was there, for example, a persistent tendency for the D-children after Test I to revert to its use in spite of their teachers' opposition? Did the O-children betray any uncertainty and indecision in substituting the new short form for the longer crutch? Or, on the other hand, did the D- and O-children drop the crutch easily, and did the R-children object often or ve-

hemently to the required use of the crutch? Evidence on these questions comes for the most part from the teachers' logs.

R-sections

Two teachers reported that none of their pupils raised a question at any time about the crutch prior to or immediately following Test III. The children who omitted it did so unobtrusively and without first taking the matter up with their teachers. The other children in one of these two sections, not having seen a shorter form and apparently not being curious as to the possibility of subtracting without the crutch, never deviated from its use, and never, even after Test III, *asked* for a shorter form. In the other of the two sections a few children noticed the absence of the crutch in the work of a repeater. They tried out the shorter form on a few subtraction examples, but with disastrous results: their answers were generally wrong. And so they were convinced of the value of the crutch and continued thereafter to use it regularly.

The logs of the other two teachers of R-sections contain no comments on the question of difficulty of abandoning the crutch. These were the teachers of the sections whose pupils were responsible for the increase in per cent of non-users of the crutch from 12.6 to 37.7 between Tests II and III (Table 13). On this account it is unfortunate that no records are available to describe their pupils' behavior in making the change to the shorter form. Still, it may not be an unwarranted assumption to infer that had the children of these sections experienced great difficulty in discarding the device, the teachers would hardly have remained silent.

D-sections

One teacher stated: "All my pupils dropped the crutch at the first suggestion (after Test I), and none of them, except one boy, had any trouble," that is, revealed difficulty at the change to the shorter form, or lost efficiency in subtracting.

Another teacher reported that even before Test I some of her pupils asked whether they could subtract without the crutch. A few actually demonstrated to their own satisfaction (and the teacher's) that they could, so that it was necessary for her to tell them to continue to use the crutch, because "I want to see how you subtract." After Test I, "with apparent ease the children dropped the crutch. Only two children used it often, though a larger number (not many) used it occasionally in written work." Still later, "I

continued to remind the children to drop the crutch. They did so without any trouble in forgetting to borrow."

A third teacher simply said, "The crutch was readily abandoned by all my pupils" after Test I.

The fourth D-section teacher wrote at length. The following statements are but a few from her especially interesting and complete log: "Several days before Test I one child asked if he had to 'mark out' the numbers, and two other children began to drop the crutch consistently." After Test I, "the children tended to omit the crutch at once. A few kept it with hard examples." Later, "the poorest of my pupils still cling to crossing out; otherwise they forget that borrowing has taken place." At the time of Test II, "the class has abandoned the crutch of their own free will. A few actually need it, yet do not want to use it"—probably because of the teacher's opposition thereto.

O-sections

Two O-section teachers reported nothing of interest on this particular problem.

The third teacher in this group said: "When they were shown the shorter form, twenty-two out of twenty-nine dropped the crutch entirely. It was surprising to me that so many were able to give it up the first time they were shown how to work without it. Compared with the work they had done formerly, in which they had used the crutch, today's work did not seem any less accurate." The next day five out of twenty-seven used the crutch, only one of whom asked for help in working without it. A week later he had ceased to use the crutch.

The fourth teacher reported: "Five of the children used the crutch after they were told that they might abandon its use. Three used it for about two days, and then dropped it when they saw the examples could be worked much faster without it. Two children wanted to drop the crutch before they were told they could." Two other children, the poorest in the grade, continued to use the crutch throughout the experiment on the advice of the teacher.

The Transition to the Short Form

For a good number of children the change from the crutch to the short form was too large for a single jump. Instead, they bridged the gap in a variety of ways. In this connection the following quotations from the log of a D-teacher are especially interesting and enlightening:

"Page 91. After board work (during which the children worked the example using the crutch, then erased and worked it again without the crutch) I gave them the page to do at their seats, urging them to abandon the crutch 'if they wished to do so.' LJ had been anxious to stop using the crutch. He finished his written work quickly, used no crutch, and got 100 per cent. Many children used the crutch on 'hard' examples, but left it off in easy ones. The class as a whole responded well to adondoning the crutch. The most common error occurs when they forget that they have borrowed. In many cases the figures in the second column were crossed out, but the '1' was not used. BW abandoned the crutch entirely of his own accord. . . . BS did not 'cross out' but carried the little '1' to the first column. She was the only child who did that. The other tendency was to abandon the '1' first, then to abandon the 'crossing out.'

"Page 91 repeated. After more explanation the class repeated page 91, and attempted to abandon the crutch. I asked KJ, 'How can you do it if you don't cross out?' He said, 'Well, but you do cross out even when you don't.' Asked to explain, he said, 'You cross out anyhow even if you don't put it down.' Another child said, 'You gotta remember.'

"My class has turned to pantomime! They go through all the motions in air of crossing out and carrying when they are at the board! It is very amusing. After I told them that they were like Indians using sign language when they did it, most of the pupils stopped the pantomime. The slow pupils continued to do it for some time. . . .

"LS, JW, and PH (all slow pupils) abandoned the crutch at my urging, but they cannot do it well. Their marks on their papers are low. In all cases they forget that they have borrowed."

Several days later: "JW is so anxious to do as I've told him. He crosses out, works the example, then erases the crosses and small numbers. . . . WD invariably subtracts the smaller number from the larger, regardless of where it is placed in the example, unless he is allowed to cross out and borrow using the crutch. He is not ready to give it up yet; his absence for a week caused his already slow arithmetic to suffer."

Table 19 shows something of the variety of intermediate steps between the crutch and its complete abandonment, as well as the extent of their use and certain personal data concerning their users.

The significance of these data should not be misinterpreted. The reactions of these children, in filling in steps before the adoption of the short form, are not correctly understood if they are viewed merely as evidence against the initial teaching of the crutch. These reactions were made, less because the crutch had been taught, than because they were necessary if the children were to be successful in subtraction. On this point we have (1) the word of the teachers that these children were really trying to get correct answers and

TABLE 19

VARIETY OF INTERMEDIATE STEPS BETWEEN THE CRUTCH AND ITS ABANDONMENT, TOGETHER WITH CERTAIN PERSONAL DATA OF THE USERS OF THESE STEPS

Pupil	IQ	MA	Pretest		Type of Partial Use on Test				
			Acc.	Rate	I	II	III-1	III-2*	III-3
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
R-Section									
VS	84	8–11	..	..	(1)	(1)	(1)	..	(1)
RH	102	9–7	20	21	(2)	(2)	(2)	..	(2)
RT	105	9–4	19	20	(2)	(2)	..	..	..
FJ	91	8–11	19	11	x	(3)	..	..	(2)
RP	108	8–0	11	19	x	(2)	..	..	..
MP	109	9–6	19	19	x	x	..	..	(2)
D-Section									
MS	104	9–0	17	16	x	x	(2)	..	(2)
BF	109	9–8	15	20	x	x	(4)	(4)	(4)
BS	111	8–11	7	10	x	x	(2)	..	(2)
O-Section									
JW	88	8–6	16	9	(2)	..	..	..	..
RS	115	9–9	20	21	(2)	(1)	..	..	..
MM	89	9–4	19	21	(5)	(5)	..	..	..
PM	83	7–0	10	7	x	(1)	(1)	(1)	(1)
EL	103	8–5	14	21	x	(2)	..	..	..
FS	97	9–3	19	21	x	(2)	(2)	..	(2)
LL	106	9–10	13	19	x	(3)	..	..	..
JE	120	10–1	16	12	x	(2, 4)	abs.	abs.	abs.
EO	87	9–2	18	16	x	x	(2)	(2)	(2)
GS	109	8–8	10	21	x	x	(2)	..	(2)

*Use of crutch forbidden.
Legend: x = whole crutch used.
..= no crutch, whole or partial, used

(1)	(2)	(3)	(4)	(5)
4		Crutch	4	4
$85_{1}4$	$85_{1}4$	used but	854	854
— 3 8	3 8	erased	— 38	— 38

seemed to be unable to do so without some visible record of the process of borrowing, and we have (2) in the test papers testimony in the form of correct answers that these intermediate modifications of the crutch were actually of service to their users. The correct interpretation of these records is that these children were moving as rapidly as they could toward abandonment of the crutch, but that for a time at least they were in need of some partial remnant of the device.

Too, these data pose questions which might be disconcerting to the opponents of the crutch, namely, if so many children could not easily and at once make the transition from the crutch to the usual short form of borrowing, what must be the difficulty of children

who do not start with this device? How confusing must this process be when no tangible evidence is given them with respect to its rationale! That this question is no idle query has already been established by the comparison of errors made on Test I by crutch and non-crutch children (Table 12, and the accompanying discussion).

4. CONDITIONS OF LATE OCCURRENCE OF THE CRUTCH

Is the crutch, once it has been given up, gone forever thereafter? Is the habit of its use "broken" in the sense of being eradicated from the nervous system? Or, does it tend to reappear, and if so, under what conditions?

The data to be presented in this section will reveal that the crutch is not abandoned *in toto*; once it has been dropped, it has not been dropped for good and all. On the contrary, the habit of using it is merely superseded or overlaid by new habits which make for greater efficiency. When for any reason these new habits fail to meet the need, the old crutch habit is revived and put to work. In this investigation three different sets of conditions were uncovered to support this generalization.

a. Effect of lack of practice

It will be recalled that Test IV was given rather unexpectedly some four months after Test III. In the meantime, the teachers, having completed Chapter III of the textbook, which deals exclusively with addition and subtraction, had gone on with Chapters IV to VI. In Chapter IV, "How We Multiply and Divide," the pupils encountered less than forty separate practices on subtraction. In Chapter V, "More About Multiplication and Division," they had less than twenty examples and problems which called for subtraction. Chapter VI, "Using the Four Processes," is largely devoted to fractions and to measurement, but it does present subtraction of the types 629 – 475, 917 – 349, 460 – 376, and \$7.34 – \$0.86. In addition to the uses of subtraction which appear in connection with these new subtraction examples the children were called upon to solve more than 125 verbal problems, of which perhaps one fourth involved subtraction.

The purpose of reporting this textbook analysis is to show that, all things considered, the children in this experiment had relatively little practice on subtraction after the conclusion of the investigation proper. The teachers reported that in teaching Chapters IV

and V, and even Chapter VI in which the new types of subtraction examples appeared, they found so many other things to teach (multiplication and division facts, simple computation in the processes, problem solving, fractions, measurements, etc.) that they were forced to neglect subtraction.

When Test IV was presented, therefore, the children were somewhat "rusty" in subtraction. They were apt to forget some of the steps in borrowing, and in order to keep their work straight, they tended to fall back upon using the crutch which they had given up some months before. Virtually all the teachers, in sending in the Test IV papers, accompanied the papers by letters in which they expressed surprise over the frequency with which the crutch appeared. They had observed virtually no use of the crutch from day to day and were wholly unprepared for its occurrence on the test papers. That the crutch actually was not used to a large extent has already been shown (see Table 14 and Figure 5, for instance), but this is more or less beside the point. The point is that, however infrequently the crutch occurred on Test IV, it occurred much more commonly than the teachers anticipated. Lack of practice in subtraction apparently had had the effect of reducing efficiency in the process, or at least of reducing confidence in the ability to borrow correctly. Under these conditions the children called the crutch "out of retirement" and proceeded to use it.

b. Effect of special difficulty of subtraction examples

The tendency was marked to use the crutch when especially difficult subtraction examples were encountered. Data on this point are to be had from the test papers for Test III and Test IV.

Parts 1 and 2 of Test III were made up entirely of familiar kinds of subtraction examples, three not requiring borrowing, and nine requiring borrowing only from tens. On Part 2 all the children were forbidden use of the crutch. On Part 2 the per cents of actual uses of the crutch to the possible uses were 0.4 for Section R,[4] 0.4 for Section D, and 3.6 for Section O. Part 3 contained twelve examples, in six of which there was borrowing from hundreds only, and in six from both hundreds and tens. Instruction had been given on neither of these last types. Promptly use of

[4] These per cents are found as in Table 14. Here, however, the figures for use of part of the crutch are combined with figures for the use of the whole crutch.

the crutch rose to 43.0 per cent, 0.9 per cent, and 8.0 per cent for the three sections.[5]

Likewise, six of the examples in Test IV were of an unfamiliar, and so of a hard, kind (the last column of examples, see page xx of the Appendix). The per cents of possible use of the crutch on the eighteen known examples (Table 14) were 12.9 (whole crutch, plus partial crutch) for Section R, 1.3 for Section D, and 6.0 for Section O. On the unfamiliar examples the corresponding figures were 16.4 per cent, 2.4 per cent, and 21.0 per cent.

c. New applications of subtraction

Problem-solving. In the textbook employed in the experimental sections the procedure followed in developing subtraction computation is (a) to explain a new step, (b) to develop skill by a considerable amount of practice, and finally (c) to "apply" the new step in problem-solving. Taught in this way, the children moved away from the crutch as they advanced from (a) through (b), but at once upon the presentation of verbal problems (c) the crutch reappeared to an unwonted extent. Such is the testimony of several of the teachers of crutch sections.

Unfortunately this possibility was not anticipated; hence no specific provisions were made to collect data on this point. However, if the teachers' observations are to be regarded as trustworthy, a new function must be credited to the crutch.

Problem-solving was hard for these children (so the teachers report). In solving verbal problems the computational skills which had been mastered so far as abstract numbers were concerned were put to new uses, or they were made to operate in new settings. Under these conditions the children resorted to measures which would hold steady their control over computation, and in the case of borrowing in subtraction they reverted to use of the crutch which served their needs admirably.

[5] It is customary to regard (1) borrowing from tens only, (2) borrowing from hundreds only, and (3) borrowing from both hundreds and tens as radically different types of examples, different enough to call for a wide spread in teaching. Thus, type (1) is usually taught first, then type (2) but only after an interval of time, and finally type (3), again after a lapse of some weeks. It is interesting to note, therefore, that the children in this experiment found much in common between the types. Section NC suffered a loss in accuracy of 25 percentage points from 86 per cent to 61 per cent; Section R, of 20 points, from 88 per cent to 68 per cent; Section D, of 18 points, from 86 per cent to 68 per cent, and Section O of 34 points, from 78 per cent to 44 per cent, in passing from type (1) to types (2) and (3) combined. Evidently, there were large amounts of positive transfer in all four sections.

Subtraction in Division. A late report of one of the O-section teachers contains another instance of regression to the crutch in the face of new applications of subtraction.

Her pupils, she says, had got well away from the crutch in ordinary subtraction by the time she introduced division. This she did by using the "long form" with single digit divisors. The new process was so hard for many of her pupils that they used the crutch to help them out, as in the example at the right. The recurrence of the device was especially apparent in the case of the slower pupils. Since they seemed to require the crutch to work accurately in division, the teacher did not forbid them its use. This was the wise thing to do.

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    3645
9) 32805
   27
    58
    54
    410
     36
      45
      45
       0
```

CONCLUSION OF CHAPTER IV

Four different problems, all of them related to the question of the persistence of the crutch, have been treated in this chapter.

1. To the general question, Were children who learned the crutch incapable of getting rid of it? the answer is a categorical No. It is safe to say, then, that to teach the crutch is not, as has been frequently asserted, to condemn children forever and inevitably to reliance upon it. As would be expected, the history of use of the crutch varied with teachers' attitudes toward its continued use. Children who were forbidden use of the crutch gave it up readily and almost unanimously, at least in dealing with examples of the type with which the crutch was taught. So likewise did a large per cent of the children to whom use or surrender of the device was made optional, and even a small per cent of the children who were told to use it regularly. On the other hand, enough pupils in the latter two sections retained the crutch to warrant decisive action on the part of the teacher to eliminate it. The data are clear that not all children will "just naturally" give up the crutch.

2. The tendency to retain the crutch needlessly and the opposite tendency, to drop it even too soon, seem to be in no way related to such factors as CA, MA, IQ, or a given level of rate and accuracy of computation. Rather, these tendencies seem to reflect certain temperamental traits, perhaps docility and lack of inventiveness on the one hand, and independence and resourcefulness on the other.

3. Not all children were able to pass immediately and directly from reliance upon the crutch to its omission. On the contrary,

some pupils found the step too large and adopted various intermediate steps, such as the use of only part of the crutch. This fact is interpreted as suggestive of the excessive difficulty children must experience when they are given from the start only the form which they are expected finally to exhibit. Under the latter instructional conditions understanding of the process and confident performance are hardly to be anticipated.

4. The crutch does not entirely disappear even in the case of children who seem to have dropped it completely. Instead, it remains below the surface, as it were, ready for use in an emergency. In this experiment three kinds of emergency were described. (a) After absence of practice in subtraction, children tend to revive the crutch at first in order to get back into the swing of the process. (b) When they meet new kinds of subtraction with multiple borrowing, they likewise resort to the crutch to find methods of attacking the unfamiliar difficulties. (c) When subtraction is found in new applications (such as in problem-solving and in long division) which by reason of their complications threaten control over skills which have been mastered, children revert to the crutch to hold computation steady and permit concentration on the new aspects of the applications. All of these recurrences of the crutch, it should be observed, serve useful ends.

CHAPTER V

SIGNIFICANCE OF THE STUDY

In as much as the findings of the investigation have been fully summarized at the ends of Chapters III and IV, they are not repeated here. Instead, this chapter is devoted to a consideration of the significance of the findings, first for the teaching of arithmetic, then for learning theory in general.

SIGNIFICANCE FOR THE TEACHING OF ARITHMETIC

The teaching of borrowing

Borrowing in subtraction has been generally recognized as difficult for children. Various ways have been tried to meet the difficulty. One way has been simply to tell the child precisely what he is to do, and then to entrust mastery of the process to huge amounts of repetitive practice. Another way has been to tell the child in terms of ones (units) and tens, or more commonly in terms of pennies and dimes, what borrowing actually involves, and then to rely on practice to secure efficient computation. The second way differs from the first in that it provides some degree of understanding of the operation, but, like the first, in the end it puts the responsibility upon practice.

In this experiment a definite break was made with traditional procedures. The assumptions were that borrowing is difficult, first because the operation means little to children, and second because the mental processes it calls for are complicated. The first assumption necessitates some kind of presentation which will make the operation sensible; the second, some tangible or concrete method of representing the mental processes involved. Both requirements seemed to be satisfied by the subtraction crutch at the right, which, so far as could be ascertained, has not been used in any modern textbook. This device, if it is to be used at all correctly, demands from the teacher an explanation in terms of the number system, and the latter, if it is clearly given, should put into the process of borrowing the intelligibility it has so sorely lacked in the past. Furthermore, actual use of the device provides for the child a clue of what is to be done, and, at the same time, a record of what has been done. In

$$\begin{array}{r} {}^{7}\not{8}\,6 \\ 3\,9 \\ \hline 4\,7 \end{array}$$

other words, it gives an objective representation of the thought processes which comprise the operation of borrowing.

The data presented in Chapter III demonstrate that the crutch met the needs of the children who were taught it. It helped them in the early stages of learning by making borrowing easier, by preventing them from loss of accuracy in dealing with familiar kinds of subtraction learned earlier, and by giving them insight into what they were doing. By its use these children were spared the period of confusion which troubled the pupils who did not have the advantage of the crutch. Moreover, the crutch served the interests both of the bright and of the dull, both of those especially capable and of those less capable in arithmetical computation, and it did these things without loss of rate of work. Still again, the crutch was found to be of value later on, even after it had been abandoned with easy examples, in enabling children to meet emergencies, such as especially difficult and untaught subtraction examples, unusual applications of subtraction, and the need for reinstating efficiency in subtraction after a period of disuse.

On the basis of these facts this particular crutch, or one like it, is to be recommended for teaching borrowing. This statement, however, needs certain qualifications. While most children may be expected eventually to rid themselves of the device once they understand the process and once they see that they can subtract as accurately and more rapidly without it—while this condition is to be expected of most children, it is not to be expected of all of them. On the contrary, many children, left to themselves, will continue to use the crutch indefinitely, from sheer force of habit. Since in the end expertness (speed) is desirable in subtraction and since this is impaired by use of the crutch, children should be encouraged to discard the device as rapidly as they safely can. The experience of the teachers of O-sections in this experiment is that little urging is necessary to get children to drop the crutch; no impassioned appeals are called for, nor threats of severe penalties.

And what if, in spite of encouragement to abandon the crutch, some children persist in using it? There will unquestionably be some such children. Do they not constitute ample reason for never teaching the crutch? Is it not unwise to expose them, as well as all the other children in the class, to a device which permanently impairs expert performance in subtraction? There are four answers to these questions. (1) The number of these children will be very small; 5 per cent to 10 per cent is a liberal estimate. (2) It

may seriously be doubted whether use of the crutch really impairs their performance in subtraction. On the contrary, they may actually need the peculiar services of the crutch to be as successful as they are. (3) No educational device is 100 per cent perfect, and to deny the value of a device because it (by assumption) does not help 5 per cent or 10 per cent is to hold to a standard that few, if any, devices could meet. (4) Finally, it may be asked whether it is not better in the end that the few persistent users of the crutch continue to use it if its use puts meaning and understanding into a process which without it would be quite senseless.

The "crutch" as an aid to learning

The subtraction device studied in this investigation has been shown to possess none of the evil consequences usually associated with the word "crutch." Instead, it seems to have been an aid or help in learning.

There are probably many other such devices in arithmetic, some known, more unknown, which are withheld from children because of the fear that they may be crutches. Research might establish that these other devices too might make learning easier without exposing children to the dangers supposed to inhere in them. Indeed, one reason for reporting the present study at such length was the hope that it might encourage such investigations, not so much by testing separately the worth of every conceivable device as by concentrating research interest on the difficulties and needs of the learner.

Varying conceptions of the purpose of school arithmetic

So far as is known, this is the first report of an experimental study of a crutch in arithmetic.[1] Research workers have not apparently regarded this as a promising field for study. One reason is that until recently we have rather generally subscribed to a view of the learning process which, if not faulty, is at least incomplete and superficial. (This view will be considered later in the chapter.) Another reason is that our thinking has been dominated by an equally limited conception of the purpose of arithmetic in the elementary curriculum.

[1] The senior author has elsewhere reported a statistical study of a fraction crutch, but this earlier study involved no control of the teaching. Rather, an analysis was made of the situation as it was found. The facts there reported concerning the persistence of the crutch agree in every detail with the findings of this investigation. See "An Evaluation of an Arithmetic Crutch," *Journal of Experimental Education,* II (1933), 5-34.

For years we have been accustomed to think of arithmetic as a "tool" subject, in which skill is acquired by relatively unvaried practice.[2] The outcome has been thought of as efficiency, and we have set ourselves the task of making children mechanically rapid and accurate in computation, and of doing this in the most economical manner possible. The first step was to ascertain what efficiency is, and this we found in the expert performance of the mature adult. The next step was to design a course of training which we might reasonably hope to yield at once the final expert product. Accordingly, we imposed upon children as nearly as we could the reactions, and only the reactions, which we observed in the adult, and we saw to it that children engaged in these activities and in these alone.

Of late we have begun to suspect that there is something wrong in our instruction. The pupils who have been subjected to it do not seem to show the expected proficiency. Some item appears to have been missing in our teaching, and that item, we are coming to believe, is *understanding*. Furthermore, we are coming to see that this element of understanding was neglected in our teaching because it escaped observation when we analyzed adult performance, or at least the worthier kinds of adult performance. As the expert adult uses arithmetic the accuracy and the speed of his performance are obvious; we did not fail to note them. What is not so obvious is the fact that the expert adult also performs intelligently, with understanding of what he is doing. The meanings and insights which make arithmetic sensible to him do not reveal themselves in ways which are readily identified. As a result, our description of adult arithmetical behavior, and so our prescription as to teaching content and method, was incomplete; our conception of efficiency in arithmetic led us to emphasize accuracy and speed in computation and to pass over the equally essential understandings and meanings.

So long as we held to the older views of the purposes of school arithmetic there was naturally no place for the kind of device which was investigated in this study. Such devices made no contribution to the efficiency to which we held. They took children away from

[2] One person who, consistently and almost alone, has opposed the "tool" conception of arithmetic is Charles H. Judd. See, for example, Charles H. Judd, "The Fallacy of Teaching School Subjects as Tool Subjects." *National Education Association: Addresses and Proceedings* (1927), pp. 249-252. Judd's opposition to the tool conception is to be found in his writings as early as 1903. See Chapter IX in his *Genetic Psychology for Teachers*.

not toward, the adult model as it was conceived: they interfered with immediate mastery of the approved behavior pattern. If learned, they had to be eradicated from the organism before performance could be accepted as efficient.

The "efficiency" which our teaching promoted was of a very restricted kind. At most it yielded skill in computing the abstract examples of the arithmetic textbook, but it left children insensitive to number outside of the arithmetic class and incapable of dealing effectively with the quantitative situations which they encountered in their extra-school lives. Moreover, it provided no basis for intelligent appreciation of, and adequate adjustment to, the myriad quantitative aspects of their later lives as adults. The "efficiency" which we produced was well enough adapted to conditions which remained unchanged; with these, mechanical habit sufficed. But the fact is that there is little in the conditions in which we use arithmetic which remains unchanged for long. For such conditions *real* efficiency must be buttressed in understanding.

Understanding as a goal

When we put understanding first and mere facility with figures second in our conception of school arithmetic, the teaching task takes on a different form. We no longer insist that the beginner's attempts shall be small-scale reproductions of the adult model. Instead, we are willing to accept almost anything the child does at first, provided that his activity shows that he understands what he is doing. No longer, for example, do we oppose counting on the fingers as an early means of dealing with groups and with number combinations, for we realize that at this time counting is necessary if the situation is to possess any mathematical or quantitative meaning at all.

As we come better to appreciate what the new goal of arithmetic instruction means for the learner, we shall almost certainly change our attitudes toward many of the devices and aids to which we have been accustomed to object as "crutches." This does not mean that we shall let down the bars entirely and adopt whatever is offered in the name of learning aids. Nor does it mean that we shall complicate learning by adding crutches just because we can find a place for them. What it does mean is that we shall take on a different attitude toward these devices. We shall not at once reject them peremptorily as obstacles to "efficiency"; rather, we shall scrutinize them with an eye to their possible contribution to understanding.

With this, the section on Significance for the Teaching of Arithmetic might be concluded, were it not that to do so would be to stop with statements which are largely negative. This misfortune arises from the fact that throughout the report the borrowing device had of necessity (because of common educational usage) to be referred to as a "crutch." Identification of the device as a crutch promptly puts one on the defensive; one's research then consists in ascertaining whether or not the given device possesses the harmful attributes usually ascribed to crutches. Moreover, one is led to view the device as if it were separate from learning, unrelated to learning and extraneous from it, but artificially introduced into the learning situation.

So long as we continue to worry about "crutches" we shall make little headway in investigating problems with respect to teaching. What is needed instead is, if not less interest in *teaching*, then more interest in *learning*. Under these circumstances we shall come at once and directly to close grips with the learning process as such. We shall actively seek to discover how children learn, what difficulties they encounter, and what conditions give rise to these difficulties. Knowing what these conditions are, we may then hope to avoid the difficulties or to erase them once they have appeared. So long as we hold to this changed attitude, we shall favor or oppose devices like the one here studied purely in terms of their effects upon learning, adopting them if they facilitate learning and rejecting them if they hinder learning. In that day the unhappy word "crutch" will be dropped from educators' vocabularies.

IMPLICATIONS FOR LEARNING THEORY

The laboratory psychologist who has his subjects memorize nonsense syllables or learn mazes does not do so primarily to discover how best to memorize nonsense syllables or to learn mazes. On the contrary, his studies are for purposes quite beyond these: he seeks to secure light on the learning process as such.

The situation need not be different in the case of the educational psychologist who investigates the learning of the school subjects. True, unlike his colleague, he has a certain "practical" responsibility, to contribute something useful for the improvement of school instruction. Nevertheless, there is no necessary reason why his contribution must stop here. Rather, there are excellent reasons why his contribution may well transcend the purely "practical" end. This is so because both the learning contents (the school subjects

and analogous matter) and the mental processes (the so-called "higher" mental processes) which he studies are of large importance in our culture. It is so also because he investigates these processes under conditions which permit them to function in the ways in which they prevail in life, exposed to all the distracting influences which commonly affect and complicate them.

What has just been said does not imply that all learning research on the school subjects is *ipso facto* sound and valuable. Rather, the implication is that the educational psychologist has the opportunity, whether or not he realizes it, to advance significantly our understanding of human learning. And such was the hope with respect to the investigation reported in this monograph. Throughout the prosecution of the study itself and throughout the preparation of the monograph it was felt that the study of the crutch is not limited in its implications to determining the usefulness or harmfulness of the subtraction device, but that it has implications for learning theory as well.

Typical experimentation on learning

The typical experiment on learning reports two kinds of measures: rate or time measures and difficulty or error or accuracy measures. These data are then graphed to show progress in learning, and the resulting curves tend to take one of two shapes, depending upon the measures which are represented on the vertical axis (see Figures 6 and 7).

Consideration of these curves and of the supporting data on rate and difficulty—consideration of these data alone has led to an oversimplification of the learning process. Fluctuations in the learning curves, to say nothing of apparent losses in efficiency, are viewed as undesirable. The crucial problem seems to be to prevent them and to manage learning so that the ultimate degree of speed and accuracy will be attained at the earliest possible instant. It is then but a step to the conclusion that effective direction of learning consists, first, in providing the learner at the start with the very pattern which is wanted at the end, and, second, of arranging practice for the perfection of this response. Learning, then, consists in doing substantially the same thing over and over again until facility and ease of performance have eventuated.

Criticism of dominant learning theories

This conception of the learning process is open to criticism at a number of points. Among other things, (1) it misrepresents the

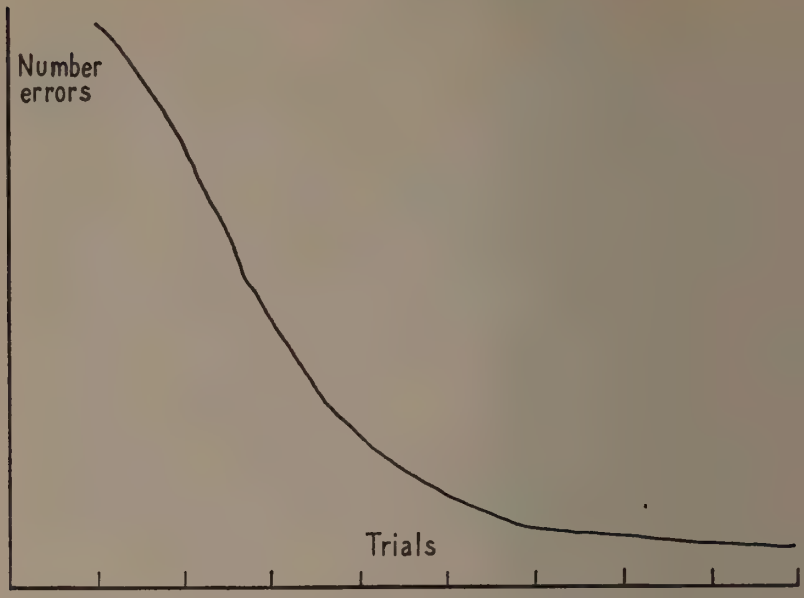

FIG. 6. Typical learning curve: reduction in errors per trial.

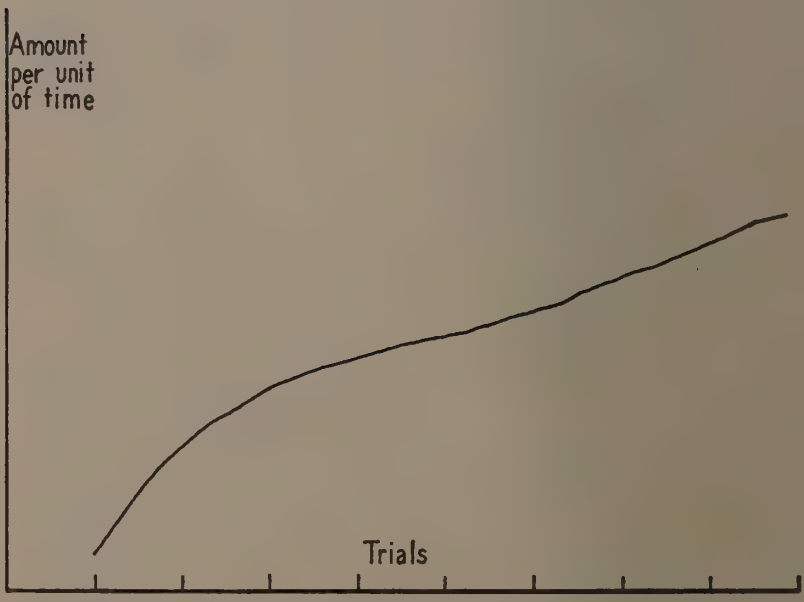

FIG. 7. Typical learning curve: amount done per unit of time.

nature of the learning process and the role of practice in learning, and (2) it contributes to false notions of economy in learning.

(1) The nature of learning. Mention has already been made of the fact that the commonly reported measures of learning are rate and accuracy of performance. But these are not the only dimensions of learning. There is a third which has been too seldom recognized. It was recognized in some of the early work on learning, as for example, in the classic study of Bryan and Harter[3] on learning to send and to receive telegraphic messages. In this study the typical learning curves appear, of course, but the curves are supplemented by explanations which are intended to account for the curves themselves. In other words, these early investigators appreciated the inadequacies of their quantitative data; they realized that the curves showed progress or non-progress *only according to the one criterion of time or of error.* But time and error data are uncertain indexes of learning. Failure to improve in these dimensions does not establish absence of learning, for learning may go on without manifesting itself in these measures. Neither does actual improvement in these dimensions tell us much; at most it can signify only that *something* has taken place, but it reveals nothing as to the nature of the happening. This it cannot do because time and error measures are not adapted to the kind of measurement required.[4]

The element in learning which has been experimentally neglected is the *process itself.* Bryan and Harter described this process in terms of the organization and reorganization of behavior, and so were able to explain their curves. Until comparatively recently, however, few others[5] have paid attention to this third dimension of

[3] W. L. Bryan and N. Harter, "Studies in the Physiology and Psychology of the Telegraphic Language," *Psychological Review,* IV (1897), 27-53; VI (1899), 348-375.

[4] This attack upon rate and accuracy measures as the sole criteria of learning is undertaken, not because the points made are new, but because they are important for any real understanding of the learning process. Dewey, as early as 1910, called attention to the distinction between process and product of learning. See John Dewey, *How We Think* (Boston: D. C. Heath and Co., 1910), pp. 53, *et seq.* See also William A. Brownell, *The Development of Children's Number Ideas in the Primary Grades* (Supplementary Educational Monograph No. 35; Chicago: The Department of Education, The University of Chicago, 1928), pp. 201, *et seq.* In this same group belongs Heinrich Kluever, *Behavior Mechanisms in Monkeys* (Chicago: University of Chicago Press, 1933).

[5] Charles H. Judd has of course persistently discussed learning in these terms, especially in his writings on arithmetic, and his position has been well demonstrated experimentally in his *Psychological Analysis of the Fundamentals of Arithmetic* (Supplementary Educational Monograph No. 32; Chicago: The Department of Education, The University of Chicago, 1927). In

learning. The reason for its neglect can only be surmised, but two possible reasons suggest themselves. One is that facts on the organization of behavior are not easily obtained, and are even less easily quantified. When we insist, as we have, that data to be experimentally respectable must be objective and quantitative, we naturally overlook or fail to report qualitative observations concerning learning in progress.[6] A second reason for the neglect of the learning process itself lies in the rather general acceptance in this country during the last two decades of a narrow S-R psychology. In accepting this psychology we limited our concern to the stimuli and to the responses which were to be connected with them and tended to disregard the means and procedures by which the appropriate reaction patterns were established.[7]

But we cannot continue longer to neglect the learning process, for to do so is to continue our ignorance and, worse, to perpetuate faulty ideas with regard to the measurement of learning.

Learning as reorganization. What one does when one learns is to attack the new problem with whatever reactions are available. These reactions are seldom if ever of the blind trial-and-error va-

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this monograph a careful study of rational counting provides data on the process or reorganization in learning. More recently, Judd has extended his analysis of learning in his *Education as Cultivation of the Higher Mental Processes* (New York: The Macmillan Co., 1936).

An especially helpful presentation of this same conception of learning will be found in the following reference: Heinz Werner, "Process and Achievement—A Basic Problem of Education and Developmental Psychology," *Harvard Education Review,* VII (1937), 353-368.

[6] The dangers of overreliance upon purely objective and quantitative data have been emphasized before. For example, consult the following references: William A. Brownell, "Educational Research on Learning," *Educational Outlook,* VIII (1934), 210-219; Kurt Lewin, *Dynamic Theories of Personality;* Karl Zener, "The Significance of Behavior Accompanying Conditioned Salivary Secretion for Theories of the Conditioned Response," *American Journal of Psychology,* L (1937), 384-403.

[7] The reader will certainly have detected the similarity between the point of view here taken with regard to learning and that set forth by Gestalt psychologists. See, for example, Kurt Koffka, *Growth of Mind* (New York: Harcourt, Brace, and Co., 1924), and Wolfgang Koehler, *The Mentality of Apes* (New York: Harcourt, Brace and Co., 1925). It is this group of psychologists who constitute the ones referred to in a foregoing paragraph as "few others" who have been interested in the organization of behavior in learning. Many of the observations of the Gestalt psychologists, made for the most part on animals during learning, would seem to be suggestive with respect to human learning. Here the Gestalt terminology is not employed, for it did not seem to be essential. As a matter of fact, the data and interpretations set forth in this monograph have been accepted by a competent connectionist psychologist as being wholly consistent with his psychological system, a circumstance which argues for the uselessness at present, of labeling experimental research as proof of this, of that, or of any other school of thought.

riety, but represent forms of behavior which have been connected previously with some aspect of the problem situation. Thus, at the very outset of learning we encounter organization of behavior, and not random or purely chance responses. The degree of success which attends the application of the first behavior organization determines (how, we need not say here) whether it will be retained and practiced, or rejected in place of another group of reactions which, like the first, have been connected with aspects, but different aspects, of the situation. As soon as some reasonably adequate reaction pattern is found, it is accepted as solving the problem. Recurrence of the problem situation sets off this adopted reaction pattern, thus insuring practice in its use. The result of the practice is increased facility in the functioning of the reaction pattern; that is, increased ease, speed, and accuracy of response.

So long as the reaction pattern satisfies all the needs of the situation as this is envisaged by the learner, that pattern will continue to be used, with ever increasing efficiency. But new needs may present themselves in the situation, or new related situations may develop new needs, and in these cases the habituated reaction pattern will not serve. Continued (repetitive) use of the old pattern will not, because it cannot, yield the newly needed behavior organization. What is required is a reorganization of behavior at a higher or more mature or more expert level. When this is achieved and first put to work, performance may be clumsy, slow, and faulty. Practice, however, removes these bars to efficiency, and behavior at the new level in time becomes rapid, easy, and accurate.[8]

The process of learning thus becomes one of the organization and reorganization of behavior or experience. The fundamental issue in learning and in the direction of learning is not practice but rather the creation of a series of reaction patterns, each of which in turn gives way to a new pattern at a higher level of organization. Danger lies, not in the absence of practice, but in possible complacency with performance at a low level. And this danger is a real one when intelligent adjustment is involved, as it *is* involved in the kind of life we should set as the goal of experience. An illustration from arithmetic will give point to these statements.

It is possible (indeed, it is a common occurrence) for children

[8] Practice too requires reorganization within the selected pattern. As that pattern becomes more and more economical, irrelevant parts of the pattern are eliminated and the necessary elements are more closely knit together. While this is true, the term reorganization is here reserved for the establishment or the discovery of essentially new patterns of behavior.

to "know" the multiplication facts only as abstract examples to be solved through counting each time they occur. Thus, the answer for 6 x 3 may not be supplied through meaningful recall, but by the slow cumbersome method of thinking "3, 6, 9, 12, 15, 18." Such "knowledge" of the fact is a *low level* reaction, entirely admirable in the early stages of learning. The danger is that the child may be content to remain at this level. When, some months later, he faces the example, 6 x 43, his solution is: "3, 6, 9, 12, 15, 18; write 8 and carry 1; 4, 8, 12, 16, 20, 24, and 1 is 25; write 25." Solutions of this kind are obviously inexpert and fraught with chances of error. What is needed is a *higher level* of "knowledge," which will function automatically but meaningfully, and which will allow concentration on the essentially new skills in the situation. The old organization of behavior is not adapted to the changed situation. Some children analyze the situation properly and, recognizing their peril, are motivated to move on to the higher level (meaningful habituation of the facts). Other children, who cannot or will not diagnose their difficulty, continue to rely on the old-order habits. With practice they may become proficient enough to satisfy the teacher's standards, but the day of reckoning is not long deferred. If it does not come with 6 x 743, it will come with 36 x 743 or with 936 x 743.

Arithmetical research on learning as reorganization. Research in the field of arithmetic has demonstrated the validity of the learning conception briefly sketched above. For example, Judd[9] in his careful laboratory study has analyzed the intricacies of the supposedly simple skill of counting and has shown the extraordinary nicety of organization which makes possible the expert counting of the finished performer. In another investigation[10] an attempt was made to discover how primary-grade children apprehend the total number of concrete objects in small groups and how they add abstract numbers. In this study it was found that a variety of procedures may be employed both with concrete objects and with abstract numbers. In the case of the former the most immature procedure is to count the objects one by one. A procedure higher in the scale of maturity is to select some smaller group within the

[9] Charles H. Judd, *Psychological Analysis of the Fundamentals of Arithmetic* (Supplementary Educational Monograph No. 32; Chicago: The Department of Education, The University of Chicago, 1927).

[10] William A. Brownell, *The Development of Children's Number Ideas in the Primary Grades* (Supplementary Educational Monograph No. 35; Chicago: The Department of Education, The University of Chicago, 1928).

larger groups, and to count onto this base the rest of the separate objects. A still more mature procedure is to add small groups, such as 2's; and so on. Thus, not only is there a variety of procedures for finding the totals (and similarly, with abstract numbers), but these procedures arrange themselves into a genetic series. Each procedure represents a level of organization which through practice becomes highly efficient. Indeed, the rapid, accurate counter, whose procedure is at the bottom of the scale, may excel (be quicker and more accurate than) the child who is at a higher level of development in finding the total number of grouped objects. Practice at one of the maturity levels does not raise the child to a higher level of reaction, but serves rather to keep him where he is, rewarding him only by increased proficiency in using his reaction pattern.

But changes do take place in the type of response which is made to a given number situation. Thus, the child eventually discards counting for more mature procedures. Investigations in fifth-grade arithmetic have also shown this process of change in response, as behavior is reorganized to serve new purposes or to satisfy new needs. In one study[11] children who were taught and required to use a particularly cumbersome device to rationalize the addition of fractions were found to have laid the device aside as rapidly as they could, even in spite of the teacher's opposition. In another study[12] children taught to divide a fraction by the common denominator method tended to abandon the use of this method in favor of the shorter and more convenient inversion method when they were exposed to it.

Both of these studies show that the adoption of one level of reaction at a low level of maturity does not necessarily condemn the child always to remain at this level. Instead, the learner, when unsatisfied, proves to be an active and persistent seeker for new modes of response. True, the counter may continue to count, and the user of the crutch may continue to use his crutch, but he will do so only so long as he is content with what his counting and his crutch produce for him. When, however, his immature procedures for any reason become unsatisfactory, he is ready to move on to

[11] William A. Brownell, "An Evaluation of an Arithmetic Crutch," *Journal of Experimental Education*, II (1933), 5-34.

[12] Thelma Tew, *Teaching Division of Fractions by the Common Denominator Method* (Unpublished Master's Thesis, Department of Education, Duke University, 1938).

the next higher reaction level, and to reorganize his behavior accordingly.

Conditions Favoring Reorganization. This reorganization, it has been said, takes place when *for any reason* the given reaction pattern is unsatisfactory. Research is badly needed at this point, in order to discover precisely what irritants have this effect, and how. A number of motivating conditions can be imagined: (1) the procedure makes for excessive errors as compared with another and superior procedure; (2) the procedure is time-consuming and holds its user to a low score on timed tests; (3) the teacher threatens punishment if the procedure is longer used; (4) the procedure is seen to be infantile, and self-respect demands a change; (5) the procedure makes the paper look "messy," etc. In the study reported in this monograph it would have been helpful if interviews could have been held with children to ascertain the reasons both for sudden abandonment of the borrowing device and for unduly long dependence upon it.

It should be clear that the present study relates directly to the conception of learning as the organization and reorganization of behavior. It therefore contributes another increment of supporting data to the accumulating body of evidence from the field of arithmetic.[13] In this study the crutch children were directed to organize their borrowing procedures according to a certain pattern (meanwhile the non-crutch children were organizing their procedures according to another pattern). The results of this instruction and of the changes in instruction have already been fully reported. At this point the results are again cited only to verify the statements made in the preceding paragraph; the crutch children, merely because

[13] All of the arithmetic research thus far cited is the work of Charles H. Judd, of the writer, or of their students. Research citations have been so restricted only because the investigations of these individuals have borne directly upon the problem of learning as reorganization. Nevertheless, this is far from meaning that other research is irrelevant or possibly contradictory. On the contrary, other arithmetic research on learning seems best to be interpreted in the light of this conception of learning. For example, the error studies of Brueckner (fraction computation), of Buswell and John (computation with whole numbers), of Burge (multiplication), of Edwards (percentage), of Grossnickle (division), of John (problem solving, division), of Norem and Knight (multiplication facts), and many others reveal evidence of difficulty arising from inability to take on new forms of response, or new organizations of behavior. The Edwards study on percentage (A. S. Edwards, "A Study of Errors in Percentage," *Twenty-ninth Yearbook of the National Society for the Study of Education,* pp. 621-640) is especially enlightening in this connection. Besides reporting error classifications (which constitute the main exhibit in most error studies), Edwards attempted to go behind these errors and to locate their causes.

they learned the crutch, were not permanently bound to its use. On the contrary, for one reason or another, most of them, some of them with surprising quickness, reorganized their borrowing behavior at a new and higher level. Those who did not do so apparently had not as yet been sufficiently motivated to make the change, which is to say, the penalties from use of the crutch (such as slow rate of work) were still not strong enough to compel reorganization.

Conflict with other learning theories. The theory of learning as the continuous reorganization of behavior is thus seen to be in conflict with the learning theory now dominant in American education. According to this latter theory no reorganization appears to be necessary in learning. The learner is at once provided with the final reaction pattern, so that there is no need for changes except for changes within that pattern itself, made to bring about greater efficiency. This theory leads us to attend to the finished performance of the one who has completed learning rather than to the inadequate efforts of the beginner and to his difficulties in achieving the final reaction pattern. It makes us impatient with his slow progress and incapable of supplying the direction he requires. Moreover, it predisposes us to misinterpret mere increase in efficiency as evidence of sound learning, for improvement in rate of work, in accuracy of work, or in both may mean no more than the attainment of mechanical control over the reaction pattern given the learner. This last danger is especially serious in all types of learning in which the verbal element is prominent, since it is comparatively easy to memorize the language without grasping the idea supposedly embodied in the language. Thus, the child (or a parrot) can memorize "three and two are five" for $3 + 2 = 5$. If the language pattern is given to the child at the start of his learning, his improvement may be rapid indeed, but the result of that improvement will be the ability to produce on demand the appropriate sounds (or the noises), not the ability to use the idea in an intelligent way.

In types of learning which should result in understanding—and practically everything learned in school should be of this kind—the final reaction pattern *cannot* be given at once. The meaning of "three and two are five" does not come quickly from saying the words over and over or from reading and writing the corresponding symbols. It develops gradually, over a long period of time, and it develops in extent and depth to the degree that experiences with it are varied and so require the continuous reorganization of behavior patterns. It is for these reasons that we should not uncriti-

cally accept the dictum, "Other things being equal, form no habits which must later be broken." If we do so, we tend to forget the qualifying phrase "other things being equal," or we minimize its importance and assume that *no* habits should be formed if those habits will later be put aside. As a consequence, we seek to secure a spurious kind of economy in learning, an economy which in the end proves to have been very uneconomical.

(2) Spurious economy in learning. Eagerness to establish at once the reaction finally desired makes us prone (a) to deny to children aids and helps which do not obviously contribute to the end product, and (b) to misconceive the function of reaction patterns which may subsequently be outgrown.

(a) *Denial of needed aids.* The first source of spurious economy just mentioned needs little discussion. The director of learning (teacher or psychologist) whose main concern is with the final reaction pattern is hardly apt to invent or to suggest devices of the kind studied in this investigation. He is inclined to doubt that such devices can contribute anything to learning, for obviously what is done in using the devices differs considerably from what is finally to be done. Moreover, he is fearful that these devices, once learned, will permanently block the achievement of the final pattern. For these reasons, on purely a priori grounds he is suspicious of anything that looks like a "crutch," and he objects to exposing the learner thereto.[14]

Under the influence of these doubts, fears, and suspicions (however groundless) we are probably withholding from children learning aids of which they stand in great need. The reader can hardly have studied the evidence presented in this monograph and still question whether the learning device was of service to its users. Whether this device is the best possible one by means of which to teach borrowing is beside the point: it *did* make learning easier and more sensible. There must be many, many other points in learning arithmetic (and the other school subjects) where children are needlessly floundering at present because we fail to give them the aids they require. It is poor economy indeed to disregard their needs and to insist that they master the one reaction pattern we provided

[14] Of course if in spite of him the learner hits upon an aid which proves beneficial, the director of his learning then has a new term for the aid. Now it is not a crutch, but something else. In other words, the aid is evaluated one way from the standpoint of prediction, and quite another way after trial. The point to be noted is that the view of learning described above is essentially static and infertile: there is nothing dynamic about it, to force us to *seek* ways to help the learner.

them, or master none at all. Sound economy suggests quite a different approach: to study the child's first attempts in the learning task and to guide him from that point with aids which will facilitate learning and make possible the ultimate achievement of the final pattern.

(b) *Misconceptions concerning the functioning of aids.* Then, too, when one is concerned exclusively with the final reaction pattern, one fails to evaluate the true economy that may reside in temporary learning aids—fails to evaluate this economy properly because of failure to recognize it at all.

The economy of temporary learning aids may be illustrated by reference again to the study of the borrowing crutch. In the first place, the borrowing crutch enabled its users to bridge the gap between familiar kinds of subtraction without borrowing and the kind newly presented. For the crutch children there was little of the early confusion which was experienced in large measure by the non-crutch children. The behavior of the latter children revealed almost complete disruption or disorganization of reaction patterns previously learned. Not only were they unable to master the new process of borrowing in economical fashion, but they lost control over kinds of arithmetical behavior which they had already mastered. By contrast, the crutch children moved easily and smoothly into the new process. The practice of changing objectively the minuends of borrowing examples marked these new examples off from the nonborrowing or familiar kind. The mere fact that the noncrutch children later on became equally proficient in subtraction with borrowing should not be permitted to conceal the uneconomy of their initial period of confusion.

In the second place, the laying aside of the crutch habit in connection with familiar kinds of borrowing does not mean that that habit had no further use. This could have been so only if the crutch habit had been eradicated from the learners' organisms. But it was not so eradicated. Instead, it remained in their organisms and was susceptible to recall under unusual circumstances. Under these conditions the crutch habit reappeared and proved its value from the standpoint of economy (1) by facilitating recovery of skills which had suffered from disuse, (2) by providing a means for dealing at least tentatively with more advanced and more complicated learning of related kinds, and (3) by enabling children to disregard subtraction while they attacked the more troublesome features of new situations.

Nor is it probable that the economy of the learning aid was limited to those occasions when its use was revealed in observable form. The value of counting, to use another example, is not restricted to the times when we actually count separate objects aloud or silently. On the contrary, when we do not find totals in this way, but form groups and add these groups instead, the ability to count still functions by providing a background of meaning for what we do. If it does no more, it probably contributes to our confidence that we *can* arrive at correct totals, by counting if necessary. But it probably functions also in still more subtle ways. Our understanding of the different numbers at least in part depends upon their differences when counted, and it is not impossible that the correct and precise use of numbers is based upon the stimulation of the counting patterns which are to some degree responsible for the knowledge of numbers as such. So, similarly, the borrowing crutch may well have continued to contribute to success in subtraction on the part of its users long after the period of the experiment and after the crutch had been officially abandoned.

It is a fallacy to assume that reaction patterns do not function merely because we cannot positively isolate evidence of their operation. Yet, this fallacy is one which seems to be committed by those who advocate the early adoption, the premature adoption, of the final reaction pattern. And this fallacy in part arises from failure to view learning in terms of reorganization and therefore from the tendency to oversimplify the process of learning and the complexity of the ultimate product of learning.

The place of aids in directing learning

It would seem reasonable that some clue or hint or suggestion or device should be given to the learner whenever he is asked to take a step which is beyond his present powers. The existence of difficulty, whether conceptual or emotional, means that learning is blocked by some kind of deficiency: unreadiness to advance, wrong attitude, or whatnot. But it is the *nature* of the deficiency, and not merely the *fact* of deficiency, which is important for the sound direction of learning; for the nature of the deficiency determines the kind of direction to be furnished. As a matter of fact, the learner may be himself unaware of any deficiency if he can meet the problem situation in some way which extricates him from his dilemma. In this case, it is the adult, and not the child, who

recognizes the deficiency. In other cases, however, the child is only too aware of his deficiency.

Be that as it may, in all cases recognition of deficiency is possible only to the observer (child or adult) who knows that there is a more mature, a more expert kind of performance. Difficulty is then experienced by the child who feels himself incapable of moving on to the next higher or to the final level; and it is noted by the adult who sees the discrepancy between the child's clumsy and uneconomical performance and the performance ultimately desired. As pointed out before in this monograph, little is accomplished by entrusting progress to continued practice; practice cannot produce the needed reorganization of response.

If the child is to use intelligently a conceptual system (like arithmetic) he must master that system through the exercise of intelligence. This is not equivalent to saying that he must master the *words* in which that system is couched. On the contrary, he must gain control over the ideas and meanings before or together with control over the verbal terms and formulations. The temptation is, however, to allow, even to encourage, purely verbal mastery.

On occasion, the advance step to be learned in the conceptual system calls for an understanding which the learner does not possess. In such circumstances what is needed is some kind of tangible or perceptual representation which will reveal concretely the intrinsic character of what is to be learned. This is precisely the function which was served by the borrowing device: it made clear what was involved in the new process. Verbal explanation, such as that given in the NC-sections, did not accomplish the purpose.

What was found in this study in respect to the usefulness of the borrowing device would seem to have general significance for the direction of learning. Abstract and conceptual learning present difficulties at times which cannot be surmounted effectively on a purely verbal basis. Indeed, at such times, exposure to a verbal explanation may defeat its own ends, for the learner can master the words so much more easily than he can master the meanings. Whenever this danger is imminent, verbal formulations should be withheld until the fundamental understandings have been developed, largely through learning activities of a nonverbal variety. Then when the basic meanings have been acquired, the verbal account and explanation should be presented as a means of summarizing and organizing the reaction pattern at the last and most abstract level.

APPENDIX

PAGE

PRETEST ii

GENERAL INSTRUCTIONS TO TEACHERS iii

TEST INSTRUCTIONS v

SPECIAL INSTRUCTIONS, Section NC vii

Section R viii

Section D x

Section O xiii

TEST I xvii

TEST II xviii

TEST III xix

TEST IV xx

Name..... School.....

Look at the sign. Then add or subtract.

Do the examples in order; (1), then (2), then (3), and so on.

Work carefully, but do not check.

(1)	68 + 49	(2)	78 − 36	(3)	$4.11 0.07 +2.40	(4)	7 5 6 + 7
(5)	976 − 473	(6)	158 − 65	(7)	53 + 27	(8)	365 − 62
(9)	369 − 43	(10)	350 + 402	(11)	5 6 8 4 + 9	(12)	139 − 45
(13)	346 + 537	(14)	174 − 84	(15)	757 − 144	(16)	54 43 + 60
(17)	29 37 + 15	(18)	805 − 303	(19)	342 208 + 135	(20)	$6.95 −5.95

GENERAL INSTRUCTIONS TO TEACHERS

The Problem. To discover the effects of exposing children to a particular "crutch" in subtraction with borrowing; specifically, to ascertain (a) whether or not the "crutch" is an aid to learning and (b) whether, if learned, it will be readily abandoned, and (c) if so, by whom.

Features of the Experiment:

1. *Experimental Schedule.* All instruction and testing is to follow with a variation of not more than two days an experimental schedule to be agreed upon, and to be administered by a co-ordinator, who will be ..... When decisions are to be made, this co-ordinator should first be consulted, and if necessary, later Dr. Brownell. Such decisions must of course be uniform and apply to all sections.

2. *Records.* Five types of record are to be used.
   a. *Tests,* three in number (later increased to five), are to be prepared at Duke University and to be scored there. (The test papers will of course first be available to teachers.)
   b. *Practice pages,* two in number, pages 86 and 95 of the textbook. The pupils' papers are to be scored at Duke University, after first serving teachers' purposes.
   c. *Instructional log,* a sort of experimental diary, to be kept by each teacher as a record of her experiences during the experiment. Entries made should include (a) the exact date on which each of the ten Outline Items was introduced; (b) observations (Sections R, D, and O) of the ease with which the crutch is taught and then abandoned; and (c) other memoranda which may be helpful in interpreting the experimental results.
   d. *Interview and conference data,* to be collected by Duke University students after, and perhaps during, the experiment.
   e. *Personal data concerning pupils,* covering such items as CA, IQ, schoolmarks, etc.

3. *Subtraction Method.* Only the method of Subtractive Decomposition as explained on page 82 of the text is to be taught

4. *Home Work.* None should be assigned, at least none which can in any way affect the experimental data. Practice in addition would of course be proper to assign for home work.

5. *Checking or Proving.* The text calls for checking and proving answers. This may constitute a handicap for the crutch pupils. Hence, teachers should make appropriate comments under *2c* above.

6. *Repeaters.* On the first test papers returned to Durham, write the word "repeater" after the name of each child who is spending his second or third year in the grade.

7. *Excessive Absences.* When a child has been absent so often that his record should be disregarded, please write this information on the first of his papers where the absences have important effect.

8. *Papers to Durham.* In all, five (later seven) sets of papers will be sent to Durham as soon as they can be collected for all four sections. These include the papers for the Pretest and for Tests I and II, and also for Practice Pages I and II (*2a* and *2b* above). (The test papers for Tests III and IV were also sent in later on.) The experimental logs will be collected later.

TEST INSTRUCTIONS

1. *Measures.* The method of administering the tests has been devised in order to secure three types of measure, namely, *rate, accuracy,* and *use or nonuse of the crutch.* The directions which follow apply to all tests and to all four experimental sections. Please refer to the directions each time you are scheduled to give a test.

2. *Rate Measures.* The following instructions hold for all tests except Test III. For directions for giving this test (Test III) see paragraph 5 below.

   a. Pass out the test sheets, face downward.

   b. At a signal have the pupils turn the papers face up and enter the information called for (name, school, etc.), turning the papers face downward once more as soon as they have supplied the required data. Wait until all have finished.

   c. Explain that an arithmetic test is to be given (but say nothing about its purpose or about the fact that the papers are to be sent to Durham). Explain further that the pupils are to work at their usual rate, as fast as they can go consistently with correct work. Make clear that it is better to have a smaller number of correct answers than a large number of wrong answers. At this time *say nothing whatsoever about the crutch,* whether it is to be used or not. Try to avoid answering directly any question concerning its use or nonuse. Also, tell the children *not to check* their answers, but to do each one carefully the first time so that no checking will be needed.

   d. At a signal have the pupils turn over the papers and begin work at once.

   e. *At the end of exactly seven minutes* have the pupils draw a circle around the example on which they are then working. They are to do this promptly, and then keep on with their work.

   f. Take up all papers when all except the slowest 10 per cent or 15 per cent have finished.

   g. Obviously, the rate measures are to be obtained from *e.*

3. *Accuracy Measures.* The papers will be scored for use in this experiment at Duke University. The scores will be the number of correct solutions. You may of course score the papers for your own purposes before forwarding them, but if you do, please make no marks or changes which will confuse the Duke scorers.

4. *Use or Nonuse of Crutch.* Information on this point will be obtained directly from the papers. You may disregard this matter entirely, for it concerns the investigation itself rather than your own instruction.

5. *Instructions for Giving Test III.* Note that this test consists of three parts, numbered 1, 2, and 3. The twelve examples in each of Parts 1 and 2 are of the familiar type; they require borrowing from tens' place only. The twelve examples in Part 3 are new: the first row of six call for borrowing from hundreds' place (not tens'), and the second row of six, from both hundreds' and tens' places. The purpose of this test is to see how well the pupils can do with unfamiliar kinds of subtraction.

   a. Pass out the papers, face downward.

   b. At a signal, have the children turn the papers over, and *promptly* fold them on the second heavy line in such a way that they will be unable to see the second and third parts while working on Part 1.

   c. Start the pupils on Part 1, and have them mark the example on which they are working at the end of three minutes. Then have them continue with the test until all but 10 per cent or 15 per cent have finished. In connection with Part 1, say nothing about the crutch, and parry any questions which arise concerning it. When the children have completed Part 1, have them wait until you start them on Page 2.

   d. Part 2. Administer Part 2 exactly like Part 1, except that you are to *deny* all children the use of the crutch, warning them that deductions will be made for examples which show use of the crutch on the test sheets. After they have taken Part 2 (the time signal being given after three minutes), have the children cover Parts 2 and 3 by refolding their papers.

   e. Part 3. Tell the children that you want to see how well they can do on new kinds of examples. Tell them that they have never worked examples like those in this part, and that you do not expect them to get all of them right. Then proceed as with Parts 1 and 2. Give the time signal after three minutes.

   f. Collect all papers as usual.

6. *Forwarding.* As soon as is practicable, the co-ordinator will collect all papers and return them to Dr. Brownell.
All tests will be furnished by the investigator in mimeographed form.

SECTION NC

Outline Item

1. Text, 79-81

   Teach these pages exactly as required by the text, and do it thoroughly, for we do not want to start the investigation proper until we are sure that the children (a) have command of the S-facts and (b) understand and can use the process. For this reason, supplement the text practice as much as seems to be necessary, keeping, however, to the investigation schedule as agreed upon.

2. Test I

   See the special page on Tests. The papers are to be collected and sent to Durham, after you are through with them. They will not be returned.

3. Text, 82-85

   Teach exactly as provided in the text. Teach the method of Subtractive Decomposition (SD) as explained on page 82 and skip pages 83 and 84. If the crutch appears on any papers, question the child as to where he learned about it and discourage use. Assign at least the amount of practice provided on pages 82 and 85, and as much more as is needed and is consistent with the schedule.

4. Practice Page I, Text, 86

   Teach according to the text, using the method of SD as on page 82. Use Exs. 1, 6, 11, 16, and 21 for illustrative work at the blackboard or as supervised seatwork. Assign Exs. 2-5, 7-10, 12-15, 17-20, and 22-25 for individual work, the examples to be copied and handed in. These papers (when you are through with them) are to be sent to Durham. Be sure pupils' names and your name appear on the papers so that they can be readily identified.

5. Text, 87-90

   Teach according to the text to the bottom of page 90, supplementing as needed and as is consistent with the experimental schedule. Discourage all use of the crutch. Assign at least as much work as is called for in the text.

6. Test II

   See special page of instructions on tests.

7. Text, 91-95½

   Teach as before to the middle of page 95. Continue method of SD and discourage all use of crutch. Assign as seatwork at least as much practice as the text affords.

8. Practice Page II, Text, 95½

   Require that Exs. 1-18 be copied and handed in. These papers are to be sent to Durham. You need not score them, except as it suits your purposes. (See Item 4 above.)

9. Text, 96-100

   As before with text pages. Method of SD; discouragement of crutch.

10. Test III

    See special page on Tests.

Remember to keep fairly closely to the schedule; do this by calling the co-ordinator rather frequently. Adjustments may be made as needed, but must be made for the group as a whole.

SECTION R

Outline Item

1. Text, 79-81

   Teach these pages exactly as required by the text, and do it thoroughly, for we do not want to start the investigation proper until we are sure that the children (a) have command of the S-facts and (b) understand and can compute accurately in S. For this reason, supplement the text practice as much as seems necessary, keeping, however, to the investigation schedule as agreed upon.

2. Test I

   See special instructions on Tests. The papers are to be collected and sent to Durham when you are through with them. They will not be returned.

3. Text, 82-85

   Teach the method of Subtractive Decomposition, as explained on page 82. This means that you will omit pages 83 and 84. However, here you are to introduce the crutch, and this is not shown in the text. The text illustrative example $(65 - 28 = 37)$ should be placed on the blackboard as shown at the right, and the usefulness of changing the figures should be carefully explained. In the same way use the following four examples at the blackboard and/or as supervised seatwork, to show how the crutch works and what its advantages (if any) may be. Do not assign the textbook examples until you are sure that your pupils understand exactly how to use the crutch, how to change the figures, etc. If the extra illustrative examples at the right are not enough, devise others. In assigning the textbook examples on page 82, tell the pupils that they *must* use the device. Assign extra examples of the kind on page 82 if this seems desirable, and ***require the use of the crutch.***

$$\begin{array}{r} {}^{5}{}_{1} \\ \not{6}\,5 \\ 2\,8 \\ \hline 3\,7 \end{array}$$

Extra illustrative examples

71	83	54	62
16	46	39	25

Skip pages 83 and 84.

Page 85: Teach according to the method of Subtractive Decomposition; use the crutch, and require the crutch to be used and shown on the papers. Work through the two text illustrative examples as shown at the right, and as extra examples for illustrative work at the blackboard or as supervised seatwork use the four examples below:

$$\begin{array}{cc} {}^{6}{}_{1} & {}^{5}{}_{1} \\ 474 & 9\not{0}5 \\ 226 & 49 \end{array}$$

$$\begin{array}{cccc} 752 & 671 & 833 & 366 \\ 329 & 255 & 19 & 47 \end{array}$$

When you are certain that your pupils fully understand how to use the crutch and how to write the changed and inserted figures, assign all twenty-five examples on the page (for one or two lessons) and *require the crutch to be shown on the papers.*

4. Practice Page I, Text, 86

The papers, which are to contain the examples mentioned below, are to be collected and sent to Durham when you are through with them. For this reason, the examples should be assigned with a great deal of care.

Teach according to the method of Subtractive Decomposition, using the crutch. Change the illustrative text examples as shown at the right: For extra practice, use text examples 1, 6, 11, 16, and 21, which may be written on the blackboard, or used in supervised seatwork.

$$\begin{array}{ccc} {}^{5}{}_{1} & {}^{3}{}_{1} & {}^{12}{}_{1} \\ \not{6}0 & 3\not{4}6 & 4\not{3}5 \\ 7 & 239 & 428 \\ \hline 53 & 107 & 7 \end{array}$$

When your pupils show that they understand the new kinds of S and know how to use the crutch with them assign as seatwork or as the next day's lesson the other twenty examples on this page: 2-5, 7-10, 12-15, 17-20, 22-25. In making the assignment say nothing at all about the crutch and try to avoid answering directly any question as to whether the crutch should be used. Here we begin to collect the data of real importance in this investigation. The papers are to be sent to Durham.

5. Text, 87-90

Page 87: Use the crutch with the two illustrative examples in the text, altering them as at the right. If further illustrative work is necessary, use the four examples below, either at the blackboard or as supervised seatwork. Then assign the twenty text examples for one or two lessons, and *require* the crutch. Give as much extra practice as seems necessary with this kind of S, devising your own examples. Try to stay fairly close to the investigation schedule.

$$\begin{array}{cc} {}^{1}{}_{1} & {}^{1}{}_{1} \\ \$5.25 & \$1.25 \\ 2.18 & 1.16 \\ \hline \$3.07 & \$0.09 \end{array}$$

$$\begin{array}{cccc} \$8.23 & \$7.31 & \$3.34 & \$4.62 \\ 4.16 & 5.17 & 3.15 & 4.47 \end{array}$$

Pages 88-90: Require the use of the crutch in the written solutions.

6. Test II

   See special sheet of directions on Tests.

7. Text, 91-95½

   Beginning at this point your instruction differs from that in Sections D and O, just as heretofore beginning with Outline Item 3 your instruction differed from that in Section NC. You are to continue to use the crutch in all explanatory, blackboard, and supervised seatwork, and you are to continue to require the appearance of the crutch on all assigned work.

   Page 91: Top half: Require the use of the crutch in the written solutions. Bottom half: same.

   Pages 92-95½: Require the crutch in all solutions.

8. Practice Page II, Text, 95½

   Assign the first three rows of examples (Exs. 1-18) to be done as seatwork or as the next day's lesson. Say nothing at all about the crutch. (See Item 4.) Collect the papers, and when you are through with them send them to Durham. These papers provide another important check on the use of the crutch. Assign the remaining eighteen examples as practice.

9. Text, 96-100

   Require the use of the crutch.

10. Test III

    See special sheet of instructions on Tests.

SECTION D

Outline Item

1. Text, 79-81

   Teach these pages exactly as required by the text, and do it thoroughly, for we do not want to start the investigation proper until we are sure that the children (a) have command of the S-facts and (b) understand and can compute accurately in S. For this reason, supplement the text practice as much as seems necessary, keeping, however, to the investigation schedule as agreed upon.

2. Test I

   See special instructions on Tests. The papers are to be collected and sent to Durham when you are through with them. They will not be returned.

3. Text, 82-85

   Teach the method of Subtractive Decomposition, as explained on page 82. This means that you will omit pages 83 and 84.

   On page 82 you are to introduce the crutch, and this is not shown in the text. The text illustrative example ($65 - 28 = 37$) should be placed on the blackboard as shown at the right, and the usefulness of changing the figures should be carefully explained.

$$\begin{array}{r} {}^{5}{}_{1} \\ \not{6}\,5 \\ 2\,8 \\ \hline 3\,7 \end{array}$$

In the same way use the following four extra examples at the blackboard and/or as supervised seatwork to show how the crutch works and what its advantages (if any) are.

71	83	54	62
16	46	39	25

Do not assign the textbook examples until you are sure that your pupils understand exactly how to use the crutch, how to change the figures, etc. If the extra illustrative examples given are not enough, devise others. In assigning the textbook examples on page 82, tell your pupils that they *must* use the device; that is, their papers, which will be handed in that day or on the next day, *must show* the device. Assign extra examples of the kind on page 82 if more practice seems desirable, and *require the use of the crutch.*

Skip pages 83 and 84.

Page 85: Teach according to the method of Subtractive Decomposition; use the crutch in explanations, and require that the crutch appear on pupils' papers. Work through the two text illustrative examples as shown at the right, and for extra illustration at the blackboard or as supervised seatwork use the four examples:

6_1	5_1		
474	965		
226	49		
248	916		

752	671	833	366
329	255	19	47

When you are certain that your pupils fully understand how to use the crutch and how to write the changed and inserted figures, assign the twenty-five examples on the page (for one or two lessons) and *require the crutch to be shown on the papers.*

4. Practice Page I, Text, 86

The papers, which are to contain the examples listed below, are to be collected and sent to Durham, when you are through with them. For this reason, the examples should be assigned with great care. Teach according to the method of Subtractive Decomposition, using the crutch. Change the three illustrative text examples as shown at the right: For extra practice (blackboard and/or supervised seatwork) use text examples 1, 6, 11, 16, and 21.

5_1	3_1	2_1
60	346	435
7	239	428
53	107	7

When your pupils show that they understand the new kinds of S and know how to use the crutch with them, assign as seatwork or as the next day's lesson the other twenty examples on this page: 2-5, 7-10, 12-15, 17-20, 22-25. In making the assignment, say nothing at all about the crutch, and try to avoid answering directly all questions as to whether the crutch is to be used. Here we begin to collect the data of real importance in this investigation. The papers are to be collected and sent to Durham.

5. Text, 87-90

Page 87: Use the crutch with the two illustrative examples in the text, altering them as at the right. If further illustrative work is necessary, use the four examples:

1_1	1_1
$5.25	$1.25
2.18	1.16
$3.07	$0.09

$8.23	$7.31	$3.34	$4.62
4.16	5.17	3.15	4.47

Use these examples at the blackboard, or as supervised seatwork. Then assign the twenty text examples for one or two lessons, and *require the crutch.* Give as much extra practice as seems necessary on this kind of S, devising your own examples. Try to stay fairly close to the investigation schedule, however.

Pages 88-90: Require the use of the crutch in the written solutions.

6. Test II

See the special sheet of directions on Tests.

7. Text, 91-95½

Beginning at this point, your instruction differs from that in Sections R and O, just as beginning with Outline Item 3 the instruction in Sections R, D, and O began to differ from that in Section NC. From now on you (unlike the teacher in Sections NC, R, and O) are to *begin urging the children to abandon the crutch.* You first show that the crutch isn't really necessary: the work can be done "in your head" without altering the figures. This is the way the "grown-ups" subtract. By omitting the altered figures it is possible to save time. You will not suggest penalties for use of the crutch, but will use all legitimate methods to encourage discarding the crutch.

Page 91: Top half: Say nothing about the crutch in the solution of the four problems. Bottom half: Precede the assignment of the twenty examples by instruction on the following four examples. Solve each example at the blackboard, first by using the crutch and altering the figures, then by recopying the example and showing how to subtract without altering the figures.

75	$3.64	370	584
49	2.59	52	577

For extra illustration, at the blackboard and/or as carefully supervised seatwork, use the four examples below at the right. When you are sure that your pupils see how to make the subtractions without using the crutch, tell them to work the twenty exercises for the next lesson (or divide and make two lessons). Tell them also that *they are not to use the crutch.* (But do not suggest that you will take away credit if they do.)

532	90	$5.73	$7.85
19	15	5.49	0.57

Pages 92-95½: Urge the children *not* to use the crutch in solving the problems.

8. Practice Page II, Text, 95½

Reference here is to the thirty-six examples at the bottom of the page. Assign the first eighteen examples to be handed in separately from the rest. (The last eighteen examples may well be reserved for another lesson.) With the first eighteen examples we want

to see whether the children will use the crutch, so in assigning these eighteen examples *say nothing at all about the use of the crutch.* The children may ask whether they are to use it, but try to avoid a direct answer. The papers for the first eighteen examples are to be collected and sent to Durham. If you assign the last eighteen examples for *another* lesson, then you should urge the abandonment of the crutch on them.

9. Text, 96-100

   As before, work steadily for the elimination of the crutch. Keep on urging the greater advantages of the shorter method, but apply no penalties for its use.

10. Test III

    See special page of instructions.

SECTION O

Outline Item

1. Text, 79-81

   Teach these pages exactly as required by the text, and do it thoroughly, for we do not want to start the investigation proper until we are sure that the children (a) have command of the S-facts and (b) understand and can compute accurately in S. For this reason, supplement the text practice as much as seems necessary, keeping, however, to the investigation schedule as agreed upon.

2. Test I

   See special instruction on Tests. The papers are to be collected and sent to Durham when you are through with them. They will not be returned.

3. Text, 82-85

   Teach the method of Subtractive Decomposition, as explained on page 82. This means that you will omit pages 83 and 84. On page 82 you are to introduce the crutch, and this is not shown in the text. The text illustrative example $(65 - 28 = 37)$ should be placed on the blackboard as shown at the right, and the usefulness of changing the figures should be carefully explained. In the same way use the following four examples at the blackboard and/or as supervised seatwork to show how the crutch works and what its advantages (if any) are. Do not assign the textbook examples until you are sure that your pupils understand exactly how to use the crutch, how to change the figures, etc. If the extra illustrative examples given are not enough, devise others. In assigning the textbook examples on page 82, tell your pupils that they *must* use the device; that is, their papers, which will be handed in that day or on the next day, *must show* the device. As-

$$\begin{array}{r} {}^{5}{}_{1} \\ \not{6}\,5 \\ 2\,8 \\ \hline 3\,7 \end{array}$$

$$\begin{array}{cccc} 71 & 83 & 54 & 62 \\ \underline{16} & \underline{46} & \underline{39} & \underline{25} \end{array}$$

sign extra examples of the kind on page 82 if more practice seems desirable, and *require the use of the crutch.*

Skip pages 83 and 84.

Page 85: Teach according to the method of Subtractive Decomposition; use the crutch in explanations, and require that the crutch appear on pupils' papers. Work through the two text illustrative examples as shown at the right, and for extra illustration at the blackboard or as supervised seatwork use the four examples:

6_1	5_1
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When you are certain that your pupils fully understand how to use the crutch and how to write the changed and inserted figures, assign the twenty-five examples on the page (for one or two lessons) and *require the crutch to be shown on the papers.*

4. Practice Page I, Text, 86

The papers, which are to contain the examples listed below, are to be collected and sent to Durham, when you are through with them. For this reason, the examples should be assigned with great care.

Teach according to the method of Subtractive Decomposition, using the crutch. Change the three illustrative text examples as shown at the right. For extra practice (blackboard and/or supervised seatwork) use text examples 1, 6, 11, 16, and 21.

5_1	3_1	2_1
60	346	435
7	239	428
53	107	7

When your pupils show that they understand the new kinds of S and know how to use the crutch with them, assign as seatwork or as the next day's lesson the other twenty examples on this page: 2-5, 7-10, 12-15, 17-20, 22-25. In making the assignment, say nothing at all about the crutch, and try to avoid answering directly any question as to whether the crutch should be used. Here we begin to collect the data of real importance in this investigation. The papers are to be collected and sent to Durham.

5. Text, 87-90

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$8.23	$7.31	$3.34	$4.62
4.16	5.17	3.15	4.47

Use these examples at the blackboard, or as supervised seatwork. Then assign the twenty text examples for one or two lessons, and *require the crutch.* Give as much extra practice as seems necessary on this kind of S, devising your own examples. Try to stay fairly close to the investigation schedule, however.

Pages 88-90: Require the use of the crutch in the written solutions.

6. Test II

See the special sheet of directions on Tests.

7. Text, 91-95½

Beginning here, your instruction differs from that in Sections R and D, just as the instruction in Sections R, D, and O began to differ from that in Section NC with Outline Item *4.* From now on, you are to *say nothing about the crutch.* In other words, your pupils will show us whether children will discard the crutch without being urged to do so. We will be able to see which children do abandon the crutch and which do not. However, after page 91 (bottom half) you will use only the short form in illustrative work.

Page 91: Top half: Say nothing at all about the crutch. We don't care whether the children solve the problems with or without it.

Bottom half: Precede the assignment of the two text examples by instruction on the following examples. Solve each example at the blackboard, first by using the crutch, and then, after recopying, without the crutch. Explain that older people find they can subtract without writing the little figures, that it saves time, and that perhaps they too (the children) would like to use the shorter way. Do not, however, urge them to abandon the crutch.

75	$3.64	370	584
49	2.59	52	577

For extra illustration at the blackboard and/or as supervised seatwork use the four examples at the right. Then when you are sure that your pupils see how to work the examples without the crutch, assign the twenty examples on the page for one or two lessons. Say nothing as to whether the crutch is to be used or is not to be used, telling the children (if they ask) that they may do as they wish.

532	90	$5.73	$7.85
19	15	5.49	0.57

This is important.

Pages 92-95½: Let the children do as they wish in solving the examples and problems on these pages. Do not put them under any necessity of either using or discarding the device.

8. Practice Page II, Test, 95½

Reference here is to the thirty-six examples at the bottom of the page. Divide the examples into two groups, the first eighteen for

the purpose to be described, and the last eighteen as an extra lesson. Assign the first eighteen examples to be copied and handed in that day (or the next). Say nothing about the use of the crutch, and advise neither for nor against its use if you are questioned. The papers for these first eighteen examples are to be collected and sent to Durham. Assign the last eighteen examples later, with no instructions about the crutch.

9. Text, 96-100

As before, continue to take no stand concerning the use of the crutch. Let those who will, abandon it.

10. Test III

See special page of instructions.

TEST I

Name..... School.....

Look at the sign. Then add or subtract.

Do the examples in order: (1), then (2), then (3), and so on.

Work carefully, but do not check.

(1)	73 — 49	(2)	749 + 222	(3)	$1.87 — .64	(4)	854 — 38
(5)	573 — 446	(6)	$3.69 —3.35	(7)	358 + 208	(8)	$9.82 —2.34
(9)	866 — 359	(10)	$1.63 +5.16	(11)	927 — 819	(12)	705 — 502
(13)	707 + 284	(14)	698 — 567	(15)	$3.78 — .29	(16)	91 — 56
(17)	157 — 96	(18)	$7.33 —7.25	(19)	302 53 + 404	(20)	995 — 347

Name..... School.....

Look at the sign. Then add or subtract.

Do the examples in order: (1), then (2), then (3), and so on.

Work carefully, but do not check.

(1)	723 — 19	(2)	132 + 49	(3)	\$9.51 —7.23	(4)	\$6.49 —5.42
(5)	\$3.36 +5.47	(6)	862 — 857	(7)	158 — 96	(8)	\$3.94 — .58
(9)	347 — 19	(10)	\$2.79 —2.73	(11)	426 + 353	(12)	82 — 34
(13)	\$3.76 —2.68	(14)	784 — 25	(15)	\$7.99 —2.95	(16)	412 35 + 236
(17)	569 — 34	(18)	78 + 506	(19)	61 — 47	(20)	\$7.95 —5.78

Name..... Teacher.....

I

(1) 232 −128	(2) 83 − 45	(3) 679 −263	(4) 545 −507	(5) 483 − 57	(6) 656 −319
(7) 571 −143	(8) 176 − 80	(9) 972 −769	(10) 84 − 19	(11) 853 −623	(12) 871 −759

II

(1) 465 −408	(2) 798 −364	(3) 391 −138	(4) 64 − 35	(5) 392 −284	(6) 582 −473
(7) 179 − 99	(8) 73 − 18	(9) 883 − 26	(10) 848 −502	(11) 996 −417	(12) 451 −322

III

(1) 388 − 98	(2) 539 −242	(3) 307 − 65	(4) 845 −562	(5) 729 − 55	(6) 308 −236
(7) 720 − 78	(8) 845 −196	(9) 362 − 97	(10) 871 −485	(11) 424 − 48	(12) 730 −569

Name..... School.....

Do these subtraction examples in order: (1), then (2), then (3), and so on.

Work carefully, but do not check.

(1)	713 — 404	(2)	\$5.82 — .38	(3)	734 — 225	(4)	\$9.17 —4.38
(5)	582 — 407	(6)	\$8.93 —5.27	(7)	615 — 507	(8)	3000 — 464
(9)	874 — 308	(10)	\$ 8.56 — .37	(11)	943 — 705	(12)	4078 — 396
(13)	685 — 76	(14)	\$7.82 —7.59	(15)	844 — 729	(16)	3611 —3265
(17)	853 — 146	(18)	666 — 639	(19)	872 — 24	(20)	\$7.03 —3.89
(21)	865 — 859	(22)	952 — 505	(23)	\$7.73 — .48	(24)	5020 —2238

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